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PRICES AND INCOME DISTRIBUTION

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AN INQUIRY INTO THE EFFECTS OF RELATIVE-PRICE CHANGES ON THE COST OF LIVING AND THE DISTRIBUTION OF INCOME IN SWEDEN WITH SPECIAL REFERENCE TO THE COST OF CHILDREN

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CONTENTS

1 INTRODUCTION

Changes in relative prices affect the economy in a lot of ways. It affect the allocation of resources across markets and induce the consumers to change their consumption pattern.

Here we shall concentrate on the effects of relative-price changes on the distribution of income, through the impact on the cost of living of different households.

It is an observed fact that the consumption patterns among households with different incomes vary considerably. Hence, relative-price changes imply changes in the cost of living across households.

The purpose is here to study the significance of such differences and if they have been inegalitarian during the 1970s in Sweden.

It has been observed that the variability of relative-prices have increased during the 70s and have been accompanied by a higher rate of inflation. In addition, it has been shown that an increased rate of inflation will increase the variability in relative prices and hence affect the distribution of income[1).]

The outline of this study is as follows. Chapter 2 is devoted to a general discussion of and introduction to the cost of living index and to other types of price indices. Ch. 3 defines cost of living indices and equivalence scales, based on the economic theory of the consumer. Ch.4 discusses different inequality measures and interpersonal comparisons of utility. Ch. 5 gives the empirical specifications of the demand systems and of the cost of living index numbers and equivalence scales, as well as discusses the econometric methods used in estimation. Ch.6 gives empirical results from the estimated demand systems and Ch. 7 discusses the estimated equivalence scales. Chapter 8 gives the effects on inequality and results from different inequality indexes are discussed. The distribution of total expenditures, with and without adjustments for the cost of children, are compared, and the effects on the distribution of real income from and 1978, are analysed. In Ch. 9 some simulations are also made, to analyse the distributional consequenses of price subsidies of some strategic goods, which have been pursued in Sweden and elsewhere. Ch. 10 summarises and concludes.

2 CHANGES IN RELATIVE-PRICES AND COST OF LIVING COMPARISONS

2.1 Introduction

The importance of price changes for the economic welfare was early recognised. Bishop Fleetwood in 1705 was one of the first to define a cost of living index. It was defined for a student and for his way of living. Bishop Fleetwood concludes that "since money is of no other use, than as it is the Thing with which we purchase the Necessaries and Conveniences of Life, 'tis evident, that if £3 in Henry VI. Days, would purchase 5 Quarters of Wheat, 4 Hogsheads of Beer, and 6 Yards of Cloth, he who then had £5 in his Pocket, was full a rich a Man as he who has now £20 if with that £20 he can purchase no more Wheat, Beer, or Cloth, than the other."

What the bishop had in mind was the differences in costs over time to maintain a certain level of welfare; "to purchase the same Quantity of Meat, Drink, and Cloth." Bishop Fleetwood was concerned with a student and his particular consumption pattern, but was fully aware of that changes in costs of living would vary with differences in consumption patterns or welfare levels. However, one ambiguity concerns what the consumer purchases.

Why would the student buy the same quantities of the goods? If the preferences of the student did not change it is likely that he would only like to maintain the same quantities if all prices moved in the same proportion. For instance, if the price of wheat increased more than the price of beer, it is likely that the student would substitute some wheat for beer. Of course, this has nothing to do with the consumption level (4 hogsheads of beer = 952 liters).

Therefore, if the student's stipends were to increase with the amount prescribed by the bishop, it is likely that the student would be better off through the increased beer consumption.

The Laspeyeres' type of price index used by the bishop can be interpreted in terms of the utility framework and then implies a very restrictive utility function, the Leontief-type, without substitution

Another useful insight in the bishop's book is that the cost of living index is best regarded as the ratio between two cost functions, and not as a function of price relatives[1)], which is sometimes the interpretation in the literature on index numbers.

The cost function, $c(u,p)$, depends on prices, p, and utility, u, and is increasing in u, nondecreasing in p and increasing in at least one price, which follows from the nonsatiation axiom of the consumer. Because of this property of the cost function it also follows that the cost of living index depends on the utility level:

$P(u,p^1,p^0)=c(u,p^1)/c(u,p^0)$, where 0 and 1 refer to two different situations.

Consumers with low utility levels will have their expenditures concentrated on necessities. Hence, relative-price increases on necessities will particularly hit the less well off.

Looking at households instead of consumers one can observe that households with low expenditure levels and large households have large proportions of their total expenditure spent on necessities.

To evaluate the effects of price changes on the distribution of income among households, there is a need to take into account the different needs by households with varying size. E.g., the subsistence expenditure level must be higher for a large than for a small household. Analogously, a given quantity of a good, say, 10 liters of milk, will yield a higher utility to a smaller household, simply because a smaller number of mouths should be fed. Engel(1895) took the expenditure share of food as an indicator of welfare. By his approach the concept of <u>equivalence scales</u> was introduced.

Equivalence scales, in the utility framework, are, like cost of living index numbers, ratios of cost functions, but equivalence scales are ratios of cost functions for households of different sizes, rather than prices in different situations. Engel's scales were obtained by comparing the expenditures of households of different size, at the same budget share for food. Equivalence scales then can be used to adjust, or deflate, income levels of households of different sizes

2.2 Two Different Approaches

In the empirical literature, two different approaches to the study of prices and income distribution have been used.

Basically, they differ in the way economic theory is treated or imposed. The first approach uses the expenditure patterns in the Family Expenditure Survey, together with time series price data to compute cost of living indices for different household groups, presumably partitioned according to income, or total expenditure. Examples of this approach are Assarsson(1976,1977) for Sweden and Michael(1979) for the United States. The problem with this approach is that it is based on a very restrictive utility function, allowing no price substitution.

The other approach, which will be used here, is to compute proper cost of living indices, based on the economic theory of the consumer. An example of this approach is Muellbauer(1974).

In this second approach, a complete system of demand equations is estimated and the demand functions are inverted into the indirect utility function to yield a measure of the cost of a certain utility level and of the cost of living index.

Of course, this second approach is theoretically much more attractive. However, the first approach has some advantages too. The biggest advantage is that a more disaggregated commodity breakdown can be used, which is computationally infeasible in the second approach. If there is a large relative-price variability within commodity aggregates, this is likely to affect the cost of living across households and hence the distribution of income.

On the other hand, in the first approach one is confined to the household data in the family expenditure survey. In the second approach, cost of living indices for <u>any</u> total expenditure, or utility, level can be computed.

In the sequel, we shall use the second approach, based explicitly on the economic theory of the consumer. We therefore turn to an introduction to the economic theory of index numbers: the cost of living index and the equivalence scale.

3 CONSUMPTION THEORY THE COST OF LIVING INDEX AND EQUIVALENCE SCALES

3.1 Duality and the Consumer's Problem

The problem of the consumer is to determine the consumption of different goods, given prices and income. It is assumed that the consumer does not waste any opportunities, i.e., is a utility maximiser.

We assume the existence of a utility function, i.e., the axioms of choice apply for the consumer's preferences: reflexivity, completeness, transitivity, continuity, nonsatiation and convexity. From these axioms it follows that the direct utility function v(q) exists. v is quasi-concave and non-decreasing in each of its arguments.

The choice problem of the consumer then is to maximise the utility function under the budget constraint:

$$(3.1)\quad \text{Maximise}\quad v(q)\quad \text{subject to}\quad \Sigma\, p_i q_i = x,$$

where x is total expenditure. This is the familiar problem, which can be solved by forming the Lagrangian for (3.1). From the first-order conditions the solution can be obtained as the system of Marshallian demand functions[1).]

$$(3.2)\quad q_i = g_i(x,p)\ .$$

However, the consumer's problem could also be formulated as

$$(3.3)\quad \text{Minimise}\quad x = \Sigma\, p_i q_i \quad \text{subject to}\quad v(q) = u\ .$$

(3.3) is called the dual problem, or the dual to the original problem (3.1)[2)]. In the dual problem, the consumer chooses goods so as to minimise the cost of attaining a certain utility level, u. In the original problem, the solution, q_i, was determined by x and p, but in the dual problem q_i is determined by u and p. Therefore, the solution to the dual problem, though the same, are in terms of u and p:

(3.4) $q_i = h_i(u,p)$,

which are the <u>Hicksian</u> demand functions. They state how demand are affected by prices, with u held constant. Therefore, they are also called <u>compensated</u> demand functions.

(3.2) and (3.4) can be substituted back into their original problems, which give

$$(3.5)\quad u = v(q_1,q_2,\ldots,q_n) = v(g_1(x,p),g_2(x,p),\ldots,g_n(x,p)) = f(x,p)$$

and

$$(3.6)\quad x = \Sigma\, p_i q_i = \Sigma\, p_i h_i(u,p) = c(u,p)\ ,$$

respectively. f(x,p) is the <u>indirect utility function</u>, defined by

$$(3.7)\quad f(x,p) = \max\ (v(q)\ |\ p\, q = x)$$

and c(u,p) is the <u>cost function</u>, defined by

$$(3.8)\quad c(u,p) = \min\ (p\, q\ |\ u = v(q))\ .$$

f(x,p) gives the highest possible utility given p and x and c(u,p) gives the lowest possible cost of attaining u at prices p. f(x,p) and c(u,p) are actually two alternative ways of writing the same thing. Since x=c(u,p) and u=f(x,p) both can be rearranged to give the other.

In empirical work it is often of great advantage to use the dual problem instead of the original problem. The reason is that the demand functions are very easily obtained from the cost function, and the latter can be defined to be a valid representation of consumers' preferences[3]. For the latter to be the case, the following properties of the cost function must hold[4]:

Property C1. $c(u,\theta p) = \theta c(u,p)$.

Property C2. $c(\bar{u},p) > c(u,p)$ for $\bar{u} \underline{>} u$

$c(u,\bar{p}) > c(u,p)$ for $\bar{p} \underline{>} p$

$c(u,\bar{p}) > c(u,p)$ for $\underset{j\in\bar{p}}{p_j} > \underset{i\in p}{p_i}$ for some j,i

Property C3. $c(u,p)$ is concave in p.

Property C4. $c(u,p)$ is continuous in p and twice differentiable with respect to p.

Property C1 is a consequence of cost minimisation. Property C2 follows from the nonsatiation axiom. At given prices the consumer has to spend more to be better off and, at given utility, increased prices implies that expenditures cannot decrease. The concavity property is implied because the consumer minimises cost and hence uses all substitution possibilities. C4 follows from C3.

Another, most useful, property of the cost function is known as Shephard's lemma[5]:

$$(3.9) \quad \frac{\partial c(u,p)}{\partial p_i} \equiv h_i(u,p) = q_i$$

i.e., the partial derivatives of the cost function with respect to prices are the Hicksian demand functions. Since u is not observable, the Hicksian demand functions must be replaced in empirical work. This is easily done, since $c(u,p)=x$ can be rearranged to $u=f(x,p)$, which, upon substitution, gives

$$(3.10)\quad q_i = h_i(u,p) = h_i(f(x,p),p) = g_i(x,p) \;,$$

i.e., the Marshallian demand functions, which, after specification, can be estimated with price and expenditure data.

The well-known properties of the demand functions are the following:

Property D1. $\Sigma\, p_i h_i(u,p) = \Sigma\, p_i g_i(x,p) = x$.

Property D2. $h_i(u,\theta p) = h_i(u,p) = g_i(\theta x,\theta p) = g_i(x,p)$.

D1 is the adding-up property and D2 the homogeneity property, the latter implying absence of money illusion or that relative prices are sufficient to determine demand.

Define

$$(3.11)\quad s_{ij} = \frac{\partial h_i(u,p)}{\partial p_j} = \frac{\partial g_i(x,p)}{\partial x} q_j + \frac{\partial g_i(x,p)}{\partial p_j}$$

as an element of the n by n <u>Slutsky</u> matrix S. Then the following properties must also hold:

Property D3. $s_{ij} = s_{ji}$ for all i,j, i=j

Property D4. S is negative semi-definite.

The last two properties are implied by the consistency of preferences. D4 implies, e.g., that all compensated own-price elasticities $s_{ii} < 0$.

With these properties of cost and demand functions, the general description of the theory of the consumer is completed. We now turn to the definition of the cost of living index and to the modeling of demographic variables, consistent with this theory.

3.2 The Cost of Living Index

The cost of living index indicates the relative change in income (or total expenditure) needed to be as well off after a change in prices. Indexing prices in two different situations, two points in time say, by 0 and 1, we define the cost of living index by

$$(3.12)\quad P(u,p^1,p^0) = c(u,p^1)/c(u,p^0)$$

It is instructive to compare the properties of (3.12) with some commonly used price indexes. Three candidates are Lapeyres', Paasche's and Fisher's ideal index, which are defined in terms of quantities consumed. These indices are

$$(3.13)\quad P^L(q^1,q^0,p^1,p^0) = p^1q^0/p^0q^0 \ ,$$

$$(3.14)\quad P^P(q^1,q^0,p^1,p^0) = p^1q^1/p^0q^1 \ ,$$

and

The following inequalities applies:

$$(3.16)\quad P(u^0,p^1,p^0) \leq P^L$$

and

$$(3.17)\quad P(u^1,p^1,p^0) \geq P^P.$$

These inequalities are shown in figure 3.1, in the two-commodity space. In this figure q^1 is optimal demand at prices p^1 and utility level u^1, q^1_0 at prices p^0, while q^0 is optimal demand at prices p^0 and utility level u^0, and q^0_1 at prices p^1. Hence,

$$P(u^0,p^1,p^0) = OD/OA < OE/OA = P^L$$

and

$$P(u^1,p^1,p^0) = OA/OF > OA/OG = P^P.$$

Because of the convexity of preferences, (3.16) and (3.17) always hold. However, the relationship between P^L and P^P is not possible to determine in general. Hence, P^F need not be more "true" than P^L or P^P. This is so because there are not a single true cost of living index, but it depends on the utility level. Only in the case with homothetic preferences, when consumption takes place along a ray through the origin, i.e. when $P(u,p^1,p^0)=P(p^1,p^0)$, then $P^L>P(p^1,p^0)>P^P$.

This exercise has shown two important things. First, it has focused on the relationship between cost of living indices and the commonly used fixed-weight price indices. Secondly, the cost of living index depends on u, the utility level. Hence, although consumers may have identical preferences, changes in the cost of living may differ across consumers or households and price changes therefore affect the distribution of income.

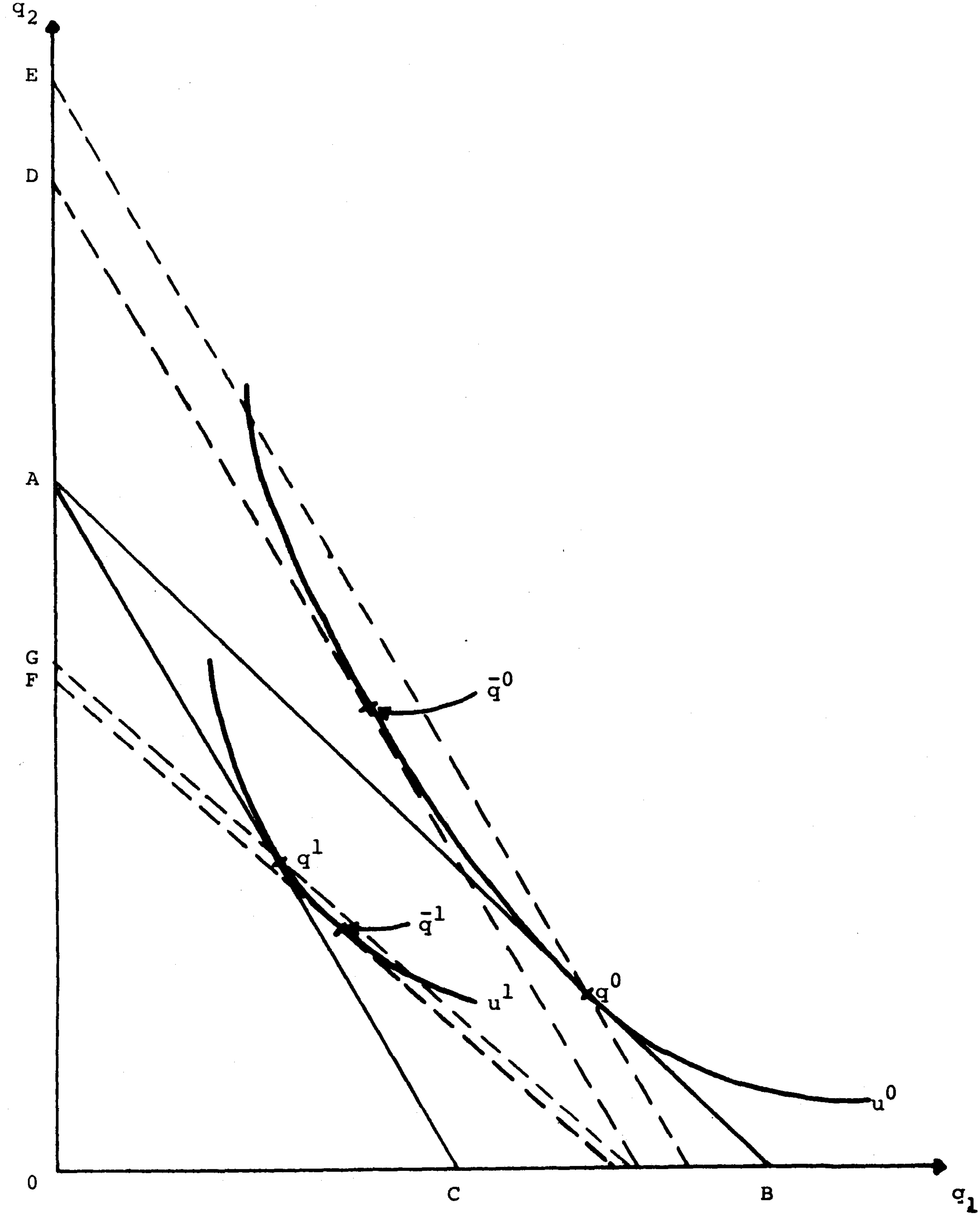

Figure 3.1. The relationships between Laspeyres´, Paasche´s and true cost of living indexes.

3.3 Cost of Living Indices for Different Households

Hitherto, the cost of living index has been defined for a single consumer. However, the object of this study is to compare cost of living indices for different households. Hence, consumers must be aggregated into households.

The reason for working with households instead of consumers are twofold, the first of which is theoretical. The household is the natural unit, since the objects consumed are very often shared within the household. The postulate in consumer theory, that the utility of one consumer is independent of all other consumer's utilities, is clearly violated within a household. In particular, the preferences of minors are probably not always well-defined, but best are expressed in terms of the parents. With the latter assumption, even minors can be incorporated into the neoclassical consumption analysis. Also, in the study of inequality it is much better with households as the objects of the analysis. The reason is that the distribution within a household probably in most cases is very even, though the observed distribution between individuals might be very uneven. An obvious example is where only one of the adults work in the labour market and one of the adults has zero market (labour) income.

The second reason is a more practical one. The consumption data are not available on the individual level, which, I think, partly is due to the theoretical arguments above. Instead, consumption data are available on the aggregate level in the national accounts or on the more disaggregated level for households in the family expenditure survey.

In the following we shall use the preferences and their representations for a household. It is then assumed that these preferences refer to an adult couple and that the preferences of these two individuals are identical.

Hence, when we study the effects of relative-price changes on the distribution of income, we restrict ourselves to adult couples, with and without children. Hence, households consisting of singles are

in the next section.
Before dealing with the problem of how to treat households with different sizes, however, we should mention the problem with price discrimination.

Differences in cost of living indexes among households with different incomes in general depend on two things. First, there must be some variation in relative-prices. Secondly, the consumption patterns must vary across households. Hence, if poor households consume relatively more on necessities and less on luxury goods, compared to affluent households, then cost of living indices across income levels will differ, if there is any variance in relative-prices of necessities and luxuries. However, there is also a possibility of differing indices if there is price discrimination; when prices depend on the utility level. Examples of such indices are given in Assarsson (1977), where housing costs depend on income. Price discrimination occurs through tax deductions for owner-occupied houses and through special subsidies paid to low-income households.

In the following, we will not take account of such differences. In comparing nominal and real income distributions, then, the price component of inequality is measured without price discrimination. However, the types of price discrimination mentioned above, present on housing consumption, will still affect the distribution of real income, but will rather be attributed, not to the price component, but to the quantity, or income, component.

3.4 Equivalence Scales

Since we are going to analyse the effects of price changes on the distribution of household income, though not obvious, it seems reasonable to assume, that of two households with the same income, the smaller household is better off than the larger household.

The concept of equivalence scales was first associated with Engel. His equivalence scale could be written

$$(3.18)\quad w_{ih} = g_i(x_h/m_{Oh}),$$

where w_{ih} is the budget share for commodity i (food in Engel's example), x_h is the total expenditure and m_{Oh} the <u>general</u> equivalence scale, and h indexes households.

The Engel curves (3.18) could then be estimated for different households and Engel's idea was that m_{Oh} would measure the compensation needed for a large household to be as well off as a small household. Hence, if m_{Oh} is taken to be one for an adult couple, Engel compared the total expenditures of other households, at the same budget share, with the reference household.

However, Engel's scale (3.18) was generalised by Sydenstricker and King (1921) and Prais and Houthakker (1955) to take account of the different needs of different goods:

$$(3.19)\quad q_{ih}/m_{ih} = g_i(x_h/m_{Oh}),$$

where m_{ih} is the <u>specific equivalence scale</u>. As shown by Cramer (1969) and Muellbauer (1975), these scales are not identifiable from household budget data.

The basic idea with these scales were first dealt with in Barten (1964), where the specific scales were consistently built into the utility framework.

The Barten commodity-specific equivalence scales are expressed in terms of the direct utility function as

$$(3.20)\quad u_h = v(q_1/m_{1h}, q_2/m_{2h}, \ldots, q_n/m_{nh}).$$

Let $q_i/m_{ih} = q_i^*$ and $p_i m_{ih} = p_i^*$. Since the budget restriction must hold for all households, irrespective of size, we get

$$(3.21)\quad x = \Sigma\, p^* q^* = \Sigma\, p\, q$$

Then we can define the new cost function

$$(3.22)\ c(u,p^*) = \min\ (p\ q \mid u=v(q^*))$$

and the indirect utility function

$$(3.23)\ f(x,p^*) = \max\ (v(q^*) \mid p\ \ q = x))\ .$$

Taking the partial derivative of $c(u,p^*)$ with respect to p_i^*, we obtain

$$(3.24)\ q_i^* = h_i(u,p^*)\ ,$$

or, equivalently,

$$q_i = m_{ih}h_i(u,p^*),$$

i.e., the Hicksian demand functions, and, upon substitution,

$$(3.25)\ q_i^* = h_i(f(x,p^*),p^*) = g_i(x,p^*) = g_i(x,p,a)\ \ ,$$

where a is the vector of demographic characteristics, and the specific scales are functions of the demographic variables: $m_{ih} = \phi(a_h)$.

The commodity-specific scales m_{ih} can be normalised to unity for some reference household. Then the general equivalence scale is defined by where c(u,p) is the cost to obtain u for the reference household. Again normalising prices to unity in some reference situation, the general scale is

$$(3.27)\ c(u,a_h,1) = c(u,a_h)/c(u,1)\ .$$

The vector of demographic characteristics could be any kind of characteristics, like race, sex, age, place of birth, etc. In the sequel we shall limit ourselves to the study of households with different numbers of children. The reference household will be an adult couple. By household preferences we then mean the preferences of the adults, which are assumed to be identical. As will be seen, the choice of an adult couple as the reference household is necessary when (3.26) is used as a measure for welfare comparisons. Then, let the vector a_h refer to the number of children in the household. The general equivalence scale then is

$$(3.28)\ c(u,p^*,p) = c(u,p,a_h)/x^0.$$

x^0 is the total expenditure of the utility-maximising childless adult couple.

Let us try to interpret the specific scales and the general scale in this setting. Consider the Barten type of direct utility function (3.20). The specific scales have an intuitive appeal. Though there is no theoretically given sign or size of the specific scales, m_{ih}, one would like to think of them as increasing with the number of children. Suppose q_1 is the quantity of milk. For a given quantity, 10 liters a week, say, the utility for an adult childless couple would be higher than for a couple with two children, since the children would need some milk too. On the other hand, the utility from a bottle of good whisky would presumably be the same for the two household types. Hence, one would expect these scales to rise with the number of children for most goods.

If we instead look into the demand functions there is another interesting, and perhaps counterintuitive, interpretation. In $c(u,p^*)$, the shadow or quasi prices p^* for food, clothes or education will be positive and higher than for whisky and cigars. There is therefore a substitution effect due to children in favour of whisky and cigars.

On the other hand there is an income effect in the opposite direction, provided food is a necessity and whisky and cigars luxuries.

In figure 3.2 we illustrate the substitution effect due to children and the effect on the general equivalence scale. It is done in the two-commodity space. Think of $q_1^* = q_1/m_{1h}$ as milk and of $q_2^* = q_2/m_{2h}$ as whisky, and of $p_1^* = p_1 m_{1h}$ as the quasi-price of milk and $p_2^* = p_2 m_{2h}$ as the quasi-price of whisky. Let p_1 and p_2 be constant. Suppose furthermore that children do not need whisky; hence $m_{2h}=1$.

The effect of adding a child to the household is illustrated by the change of the relative-price from AB to AC, which reflects the rise in p_1^*, with p_2^* constant. Hence, the substitution effect is the move from q^* to $\bar{q}^*$, so that the demand for whisky increases. The cost of $\bar{q}^*$ is the minimum cost of obtaining the same utility level, u^0, as the reference household, and the increased cost of a child is measured by the general equivalence scale, which is $c(u^0,p^*,p)$ = OD/OA.

What is shown in figure 3.2 is that the substitution effect increases the demand for whisky, but what about the demand for milk? Since $q_i^* = g_i(x,p^*)$, we have

$$(3.29)\quad q_i = m_{ih}(a_h)\, g_i(x,p^*)$$

and

$$(3.30)\quad \frac{\partial \log q_i}{\partial \log m_j} = \delta_{ij} + e_{ij}$$

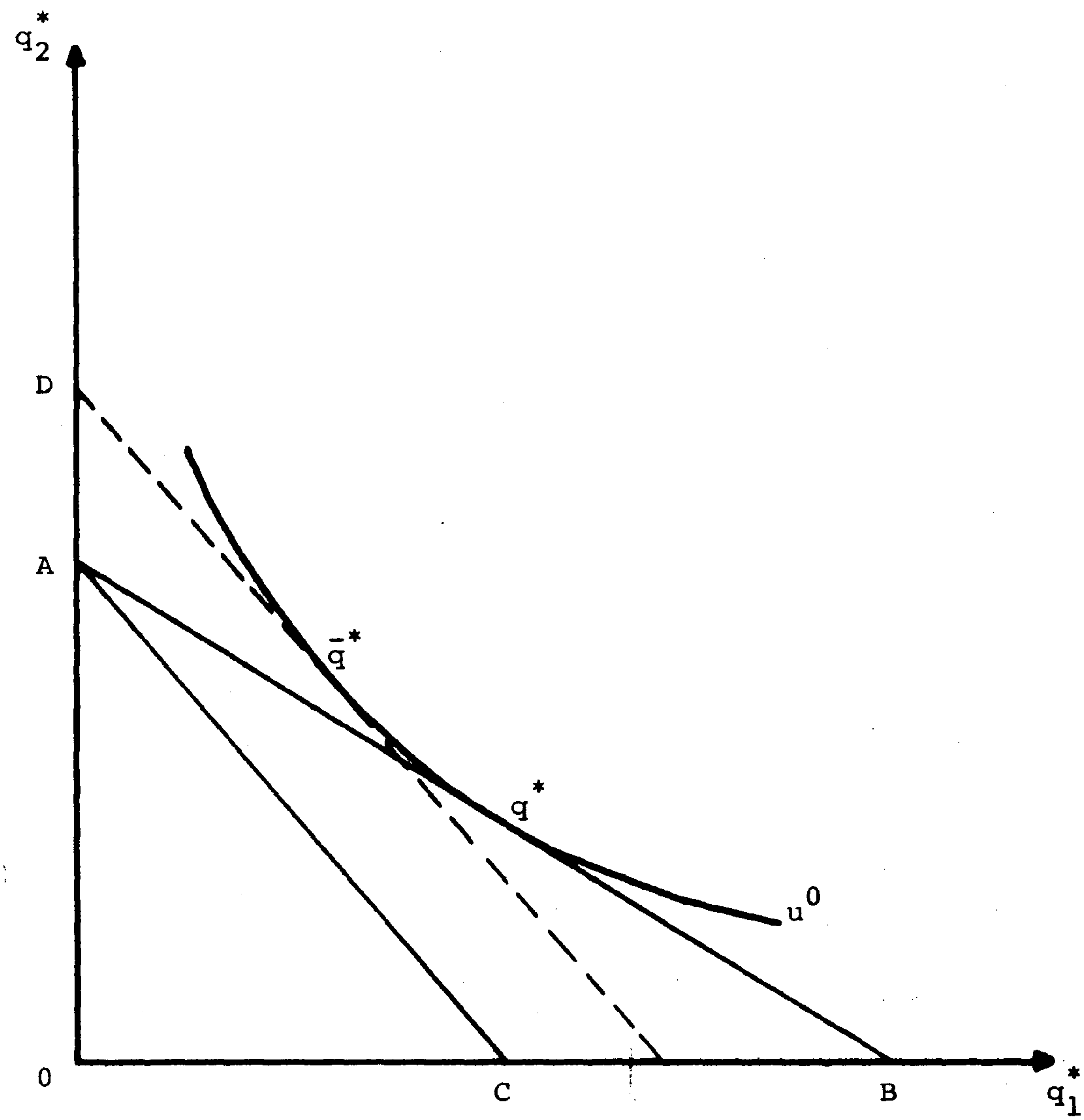

Figure 3.2. Substitution effect of children and general equivalence scale.

where e_{ij} is the Marshallian uncompensated price elasticity of demand for good i with respect to the price of good j and δ_{ij} is the Kronecker delta[6]. Hence, for the case shown in figure 3.2

$$(3.31) \quad \frac{\partial \log q_1}{\partial \log m_1} = 1 + e_{11}$$

and hence the effect on the consumption of milk could be negative, provided $e_{11} < -1$.

Perhaps such counterintuitive results have led to the questioning of the concept of equivalence scales. Equivalence scales as a basis for

welfare comparisons have been questioned by Pollak and Wales (1979). Their basic argument is that the utility function of the adult couple also should include the number of children as an argument. Hence, they would prefer a utility function like

$$(3.32) \quad u_h = \varphi(v(q^*), a_h).$$

With a_h yielding positive utility to the parents, clearly the equivalence scales previously considered exaggerate the compensation needed for constant utility and hence the traditional scales are invalid for welfare comparisons. Though the argument is basically true, it does not invalidate the use of equivalence scales, based on $v(q^*)$ only.

As noted in Pollak (1975), almost all price indices used in practice are subindices, and hence based on an implicit assumption of a separable utility function. So, the critique against equivalence scales can be raised against virtually all price indices. E.g., the consumer price index is often used to compensate different incomes, so as to make people as well off as they were before the price changes. However, since important components of welfare, such as leisure time and environmental goods are absent from the CPI, the use of CPI as a case at hand and how the subindexes in question can be interpreted. In the case of equivalence scales, the separability assumption is not an unrealistic one. It is quite reasonable to think of the decision of how to allocate total expenditure among a set of consumption goods as taken separately from the decision of how many children to bring about. Once they are brought about, there is, so to speak, not so much to do about it. Hence, the decision situation, which the equivalence scales are based on, of how to allocate expenditure on the market basket of consumption goods, <u>given</u> the number of children, is a realistic one. The equivalence scales used here are appropriate measures of the cost of children, and hence is a relevant measure for the parents in the decision between the market basket of goods and the number of children.

We have not dealt with the preferences of the children here, partly because these preferences are probably not so easily defined, in the traditional way, as the preferences of adults. However, it is reasonable that the children's utility depends on the parents' utility level from the market basket of goods:

$$(3.34)\quad u_{ch} = \Psi(u_h).$$

It is then likely that Ψ is increasing in u_h and q^* and hence decreasing in a_h, provided m_{ih} is increasing in a_h, or that the specific scales are greater than or equal to unity. This also rules out the revealed preference argument for u_h, since children (in general) cannot choose their own parents. Though the relation between u_{ch} and u_h is not explicitly modeled here, it is likely that if the notion of household preferences were to include the welfare of the children, household utility would be affected towards u_h, as obtained by traditional equivalence scales.

Finally, equivalence scales are used in social policy programmes. In such policy programmes interpersonal welfare comparisons are made and equivalence scales have been regarded as a useful tool in the guidance of policy.

4 INEQUALITY INDEXES

4.1 Introduction: Interpersonal Utility Comparisons

As discussed in the previous chapter, the application of equivalence scales in welfare theory, implies a comparison of welfare levels between households of different size. Here we shall touch upon the foundations for such comparisons[1)]. In the next section we define some commonly used inequality indexes and in the last section we discuss how inequality due to price changes can be analysed.

Certainly, the purpose of the present study is not to make comparisons of welfare in any deeper sense. We are all aware of the limits to social choice, and the approach taken here is the more practical one of making as sensible comparisons as possible. Equivalence scales and cost of living index numbers play a key role in this approach.

Quite generally the social welfare function is a representation of the preferences over social states. It could be defined over the set of individual consumption vectors, q^h, as

$$(4.1)\quad SW = \varphi(q^1, q^2, \dots, q^h, \dots, q^H).$$

φ could represent the preferences of the government authorities or could be the preferences for some individual. (4.1) is quite general but more structure can be built into SWFs by introducing individual preferences. The Bergson-Samuelson SWF is defined as

$$(4.2)\quad SW = W(u^1, u^2, \dots, u^h, \dots, u^H),$$

where $u^h = v^h(q^h) = f^h(x^h, p)$ are the utility functions of individual h. This kind of social welfare function depends on individual welfare

Following the previous analysis[2), a natural and convenient cardinalisation would be to let

$$u^h = f^h(x^h,p) = f(x^h,p) = c(f(x^h,p),p) = x^h ,$$

i.e., to let the cost function be a measure of utility, viz. the total expenditure for a utility-maximising individual. We use the common functions f and c for all individuals. However, we are primarily interested in households and hence we obtain the cardinalisation

$$(4.3)\ u^h = c(f(x^h,p,a^h),p,a^h) ,$$

where, as before, f is the indirect utility function, p is a price vector, common to all households, and a^h is a vector of demographic characteristics, in our case restricted to the number of children in the household.

We can always normalise prices to unity at some date. Then, for the reference childless couple household, our welfare measure is simply x^h, total expenditure. In other words, if prices are constant and if all households are identical, the welfare measure is equal to household total expenditure. Since prices and household characteristics are not constants, our welfare measure is

$$(4.4)\ u^h = x^h \,/\, \left(\frac{c(f(x^h,p,a^h),p,a^h)}{c(f(x^h,p,a^h),p^0,a^h)} \quad \frac{c(f(x^h,p,a^h),p,a^h)}{c(f(x^h,p,a^h),p,a^0)}\right)$$
$$= x_*^h$$

where p^0 is reference prices and a^0 is household characteristics for the reference household. The first ratio within brackets is simply a cost of living index, and the second term is an equivalence scale. Using this cardinalisation we can turn to the properties of the social

4.2 Properties of Inequality Indexes

Basing the social welfare function on (4.4), we have

$$(4.5)\quad SW = W(x_*^1, x_*^2, \ldots, x_*^H)$$

which, provided W is linear homogenous, can be rewritten as

$$(4.6)\quad SW = \bar{x}_*\ W(x_*^1/\bar{x}_*, x_*^2/\bar{x}_*, \ldots, x_*^H/\bar{x}_*)$$

where W is defined over the deviations of utility from mean utility. Let

$$(4.7)\quad E = W(x_*^1/\bar{x}_*, x_*^2/\bar{x}_*, \ldots, x_*^H/\bar{x}_*)$$

$$= \frac{W(x_*^1, x_*^2, \ldots, x_*^H)}{\bar{x}_*}$$

be an <u>equality index</u>. Then the <u>inequality</u> index I is defined as

$$(4.8)\quad I = 1 - E\ .$$

The relationship between inequality and social welfare is then defined by (4.6) and (4.8). One can easily go from one to the other. In the literature on income distribution and inequality both approaches have been applied.

The most common approach is the axiomatic approach to inequality indexes. In this approach inequality indexes are developed without reference to social preferences, and the properties of the inequality indexes are discussed from the point of view of intuition or

Proposed measures in the axiomatic approach is the variance, the coefficient of variation, the Gini coefficient and Theil's entropy measure[3).]

Various properties of these indexes have been discussed and some agreement upon some of them have come about. Dalton(1925,1926), however, emphasised that inequality measures should be based on some notion of social welfare. Hence, properties of inequality indexes can be discussed in terms of social welfare functions.

We shall discuss three properties of social welfare functions and their implications for inequality indexes. In figure 4.1 we have drawn a social welfare function for two households to illustrate these properties. The social welfare contours are drawn symmetric to the 45^{o}-ray. The consequence of this is that the condition of anonymity is satisfied. This was originally put forward by May(1952) and means that social choice should not be affected by who in particular holds certain preferences; all that matters is the ordering of the preferences.

The move from point A to point B in figure 4.1 illustrates the principle. Households 1 and 2 switch positions, hence the new situation must be on the same social welfare contour as the original situation A.

In defining the inequality index (4.8) we used linear homogeneity of the social welfare function, which simply means that a doubling of all individual utilities doubles social welfare. Though useful, linear homogeneity has been questioned by those who argue that inequality is more important in poorer societys. Linear homogeneity of the social welfare function implies an inequality index which satisfies the axiom of mean independence.

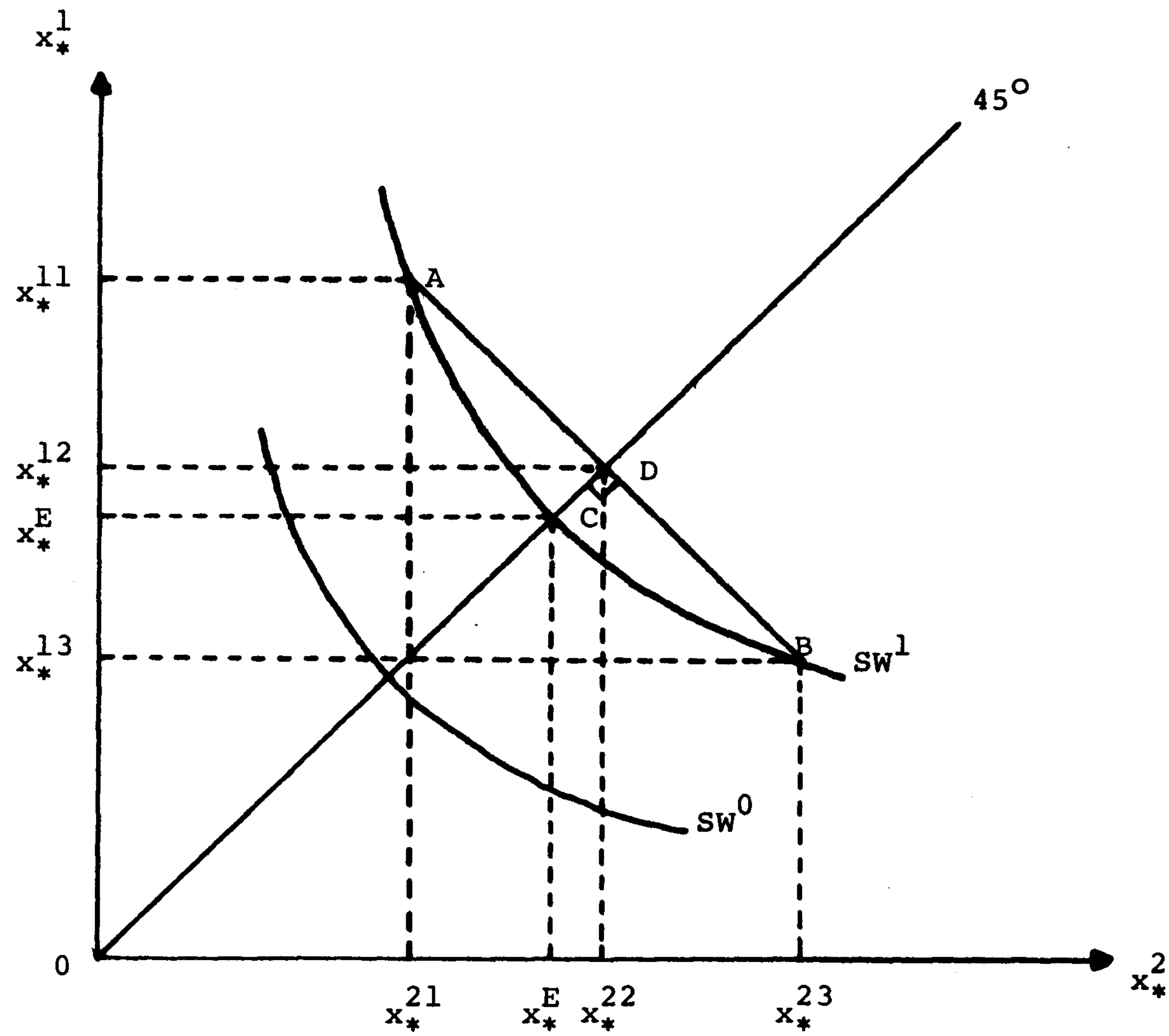

Figure 4.1. Social welfare contours and inequality indexes.

The social welfare function in figure 4.1 is strictly quasi-concave. The implication of this is that the marginal utility of income is diminishing and that an equal distribution is always preferred. This is illustrated by the fact that the point D, a linear combination of A and B, is preferred to either A or B, the relative improvement being OD/OC.

Social welfare at the equal distribution at C is equal to social welfare at the unequal distribution at A. The distribution at C is what Atkinson(1970) calls "the equally distributed equivalent income", x_*^E. This could be compared to the equally distributed actual income, at point D in the figure, to give the inequality index proposed by Kolm(1969,1976a,b) and Atkinson(1970):

$$(4.9) \quad A = 1-(x_*^E/\bar{x}_*) = 1 - \left(\sum_{h=1}^{H} \frac{1}{H} \left(\frac{x^h}{\bar{x}_*}\right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

Referring to figure 4.1 again, this index is also defined by A = 1 - (OC/OD). The parameter ε can be interpreted as the degree of inequality aversion. If $\varepsilon=0$ the social welfare contours will be straight lines, implying indifference to inequality and A=0. At the other extreme, if $\varepsilon=8$, then the social welfare contours will be right angles, implying absolute inequality aversion and A = 1.

Quasi-concavity of the social welfare function implies the Dalton principle of transfers; a transfer from a poorer person to a richer person increases inequality, unless the transfer is large enough to shift the relative positions of these persons.

Sticking to these three properties of I: mean independence, anonymity and the principle of transfers, we can look for specific inequality measures. Among those proposed in the literature, some can be dismissed. Such a candidate is the variance, since it is not mean independent. Also the Gini coefficient is rejected, provided that <u>strict</u> quasi-concavity is required. The Gini coefficient is defined by

$$(4.10)\quad G = \left(\frac{1}{2} H^2 \bar{x}_*\right) \sum_{h=1}^{H} \sum_{j=1}^{H} | x_*^h - x_*^j | =$$

$$= 1 + \frac{1}{H} - \left(\frac{2}{H^2 \bar{x}_*}\right) (x_*^1 + x_*^2 + \ldots + Hx_*^H)$$

$$\text{for } x_*^1 > x_*^2 > \ldots > x_*^H .$$

We all know the coefficient of variation, C, as the standard deviation divided by the mean and C satisfies all the three conditions above:

$$(4.11)\quad C = \left(\frac{1}{H} \Sigma \,(x_h^* - \bar{x}_h^*)^2 \right)^{\frac{1}{2}} / \bar{x}_h^*$$

The same goes for Theil's entropy measure:

$$(4.12)\quad T = \sum_{h=1}^{H} \frac{x_*^h}{\sum_{j=1}^{H} x_*^j} \log \left(H \frac{x_*^h}{\sum_{j=1}^{H} x_*^h}\right) .$$

Of course, A satisfies the required conditions, since it is based on a symmetric, strictly quasi-concave linear homogenous social welfare function.

4.3 Price Inequality

Returning now to our ultimate problem, the effects of relative-price changes on inequality can now be studied through a straightforward application of the concepts below.

(4.4) can be decomposed into

$$(4.13)\quad x_N^h = \frac{c(f(x^h,p,a^h),p,a^h)}{c(f(x^h,p,a^h),p^0,a^h)}\; x_*^h = \frac{x^h}{\dfrac{c(f(x^h,p,a^h),p,a^h)}{c(f(x^h,p,a^h),p,a^0)}}$$

where x_N^h is the nominal expenditure of an adult couple equivalent household. Then, let

$$(4.14)\quad I' = 1 - W(x_N^1/\bar{x}_N, x_N^2/\bar{x}_N, \ldots, x_N^H/\bar{x}_N)$$

be the inequality index for nominal x^h.

The effects of price inequality is then measured by

$$(4.15)\quad PI = I/I' ,$$

i.e., the relative change in inequality brought about by relative-price changes. If only absolute prices change, this measure will be unaffected, (PI = 1), since the social welfare function is assumed linear homogenous.

Measures of (4.15) with the different specifications of I and I' will be presented below. However, the specification of the individual utility framework, which is necessary for the estimations, remains.

5 EMPIRICAL SPECIFICATIONS AND DATA

5.1 Introduction

In this chapter we shall deal with the specification of functional forms for the empirical estimation. A huge literature has been occupied by this question. Stone(1954) was the pioneer in this field with the first estimation of a complete system of demand equations. Stone's linear expenditure system, which has been widely applied since then, was based on the explicit Stone-Geary utility function [1)] and hence the restrictions on the demand equations, implied by economic theory, were automatically satisfied.

Two basic approaches can be used in the specification of a demand system. The first is to specify a utility function (or some other representation of preferences) and then derive the demand equations. The other approach is to start directly from the demand equations themselves and impose the theoretical restrictions on these. In the first approach, which was used by Stone, there is the problem that demand functions cannot always be derived or that the derived demand equations become too complicated, e.g. includes nonlinearities or excessive numbers of parameters. Of course, in the second approach there is no problem of getting to the demand equations. The problem is the reverse; even if the theoretical restrictions are imposed on the demand functions, it is very often difficult to show that the particular specification is consistent with the theory of choice, i.e., that a utility function exists.

In both approaches, important work have been done. In the first approach, maybe the most important has been the specifications based on flexible functional forms. The translog model belongs to this tradition [2)]. The translog utility function, which is quadratic in logarithms, can be viewed as a second-order Taylor approximation to an arbitrary utility function. Such an approximation is usually considered accurate for small variations in the independent variables. However, even though the utility or the cost function may be a good second-order approximation, the demand functions, being first-order derivatives of the cost function, are only first-order approximations.

A well-known example of the second line of research is the Rotterdam model[3]. Though the Rotterdam model possesses a lot of the useful properties of good demand systems, a particular drawback is that it is not explicitly derived from the theory of choice. Hence, the parameters of the system cannot be used in the construction of cost of living indexes or equivalence scales in a consistent way.

A system, which seems to possess most of the desirable properties of a satisfactory demand system, has been proposed by Deaton and Muellbauer(1980a), called the AIDS (Almost Ideal Demand System). The AIDS has some attractive properties and we have good reasons for choosing this particular system.

As will become clear, the AIDS has an intuitive appealing interpretation and through the use of a flexible functional form it is a general specification of preferences, a second-order approximation of an arbitrary cost function. The AIDS can be used to test the theoretical restrictions and allows <u>perfect aggregation</u> without linear Engel curves. In addition, it is fairly easy to estimate. Since it is based on an explicit specification of the cost function, it is easy to go from the demand system to generate cost of living indexes and equivalence scales. The last property is lacking in the Rotterdam model. Compared to the translog model, the AIDS is easier to estimate. In addition, its aggregation properties are less restrictive in terms of the forms of Engel curves[4].

Estimation of the AIDS model in budget share form on Swedish aggregate time series data also produced satisfactory results as compared to other systems, as for example the Rotterdam and the translog model[5].

5.2 The AIDS

The AIDS is based on the specification of the cost function. Hence, the AIDS exploits the duality in consumer theory, as presented in ch. 3.

The AIDS cost function is defined as

$$(5.1)\quad \log c(u,p) = (1-u)\log a(p) + u \log b(p)\ ,$$

where $a(p)$, $b(p)$ are linear homogenous functions in prices p. This functional form has been called PIGLOG by Muellbauer(1975). By an appropriate normalisation, u can be scaled to lie between 0 and 1. Then, at $u=0$, $a(p)$ can be interpreted as the cost of subsistence and, at $u=1$, $b(p)$ can be interpreted as the cost of affluence or bliss. $\log c(u,p)$ is increasing in u for $b(p)>a(p)$; as u goes from 0 to 1, $c(u,p)$ increases from $a(p)$ to $b(p)$. The specific functional forms of $a(p)$ and $b(p)$ are

$$(5.2)\quad \log a(p) = \alpha_0 + \sum_i \alpha_i \log p_i + \frac{1}{2}\sum_k \sum_j \gamma^{*}_{kj} \log p_k \log p_j$$

and

$$(5.3)\quad \log b(p) = \log a(p) + \beta_0 \prod_k p_k^{\beta_k}$$

where α_i, β_i and γ^{*}_{kj} are parameters. This cost function is consistent with the theory of choice provided

$$\sum \alpha_i = 1,\ \sum_k \gamma^{*}_{kj} = \sum_j \gamma^{*}_{kj} = \sum \beta_i = 0$$

which guarantees the linear homogeneity of (5.1). The parameters α_0 and β_0 can be given a particular interpretation. If prices are normalised to unity, $\log a(p)=\alpha_0$ and $\log b(p)=\alpha_0+\beta_0$. Hence, α_0 is the cost of subsistence and β_0 the additional cost for bliss, at reference prices. Using Shephard's lemma (3.9) we find

$$(5.4)\quad \frac{\partial \log c(u,p)}{\ } = \frac{\partial c(u,p)}{\ }\,\frac{p_i}{\ } = \frac{p_i q_i}{\ } = \ldots$$

where w_i is the budget share of commodity i. Hence, differentiation of the cost function, defined by (5.1) - (5.3), gives the Hicksian demand functions in budget share form

$$(5.5)\ w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i\, u\, \beta_0 \prod_k p_k^{\beta_k}$$

where $\gamma_{ij}=1/2(\gamma^*_{ij}+\gamma^*_{ji})$. Since $x=c(u,p)$ for a utility-maximising consumer, $c(u,p)$ could be inverted to give the indirect utility function

$$(5.6)\ u = \frac{\log x - \log b(p)}{\log b(p) - \log a(p)} = \frac{\log x - \log b(p)}{\beta_0 \prod p_k^{\beta_k}}$$

In accordance with the analysis in ch.3, (5.6) substituted in (5.5) gives the Marshallian demand functions in budget share form:

$$(5.7)\ w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log (x/P)\ ,$$

where P is a price index defined by

$$(5.8)\ \log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k \log p_j$$

Maximum likelihood estimation of (5.7) will be obtained in the next chapter. (5.7) can be used to test the restrictions from the theory of choice, in particular the adding-up, the homogeneity and the symmetry restrictions:

$$(5.9) \quad \Sigma\, \alpha_i = 1, \ \sum_i \gamma_{ij} = 0, \ \Sigma\, \beta_i = 0 \qquad \text{(adding-up)}$$

$$\sum_j \gamma_{ij} = 0 \qquad \text{(homogeneity)}$$

$$\gamma_{ij} = \gamma_{ji} \text{ for all } i,j \qquad \text{(symmetry)}$$

The adding-up restriction can actually not be tested, since it is automatically satisfied in the data.

The parameters α_i can be interpreted as the budget share for a household on the subsistence level. From (5.5) it follows that $w_i=\alpha_i$ for u=0 and $w_i=\alpha_i+\beta_i$ for u=1, when prices are normalised to unity. The parameters β_i express the effect on w_i of changes in real total expenditure, log(x/P). β_i is positive for luxury goods and negative for necessities.

As shown by Muellbauer(1975,1976) the PIGLOG functional form given by (5.1) permits exact aggregation [6], if (5.7) can be generalised to

$$(5.7') \quad w_{ih} = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log (x_h/k_k P)$$

for an individual household, where k_h can be interpreted as a general equivalence scale. Defining the aggregate budget share by

$$\bar{w}_i = \frac{\sum_h p_i q_{ih}}{\Sigma\, x_h} \equiv \frac{\sum_h x_h w_{ih}}{\Sigma\, x_h}$$

and the aggregate index k by

$$\log (\bar{x}/k) = \frac{\sum_h x_h \log (x_h/k_h)}{\Sigma\, x_h}$$

the following aggregate demand functions follows:

$$(5.7'') \quad \bar{w}_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log (\bar{x}/kP)$$

If household characteristics were the same for all households, the index k would be equal to

$$(5.10) \quad k = T/H ,$$

where log T is Theil's entropy measure. If there is perfect equality in the distribution of total expenditure, k=1, then k decreases with increasing inequality, and the representative budget level increases.

In the estimations below, we shall estimate (5.7) and (5.7'') with a particular assumption of the behaviour of k.

However, to estimate the effects on inequality of changes in relative-prices, as measured by PI in (4.13), we must derive the equivalence scales and the cost of living index of AIDS.

5.3 Equivalence Scales and the Cost of Living Index

We can now use the Barten cost function $c(u,p^*)$ to define a generalised AIDS. The cost function for household h is

$$(5.11) \quad c_h(u_h,p^*) = (1-u_h) \log a(p^*) + u_h \log b(p^*)$$

where

$$(5.12) \quad \log a(p^*) = \alpha_0 + \sum_k \alpha_k \log (p_k m_{kh}) + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log (p_k m_{kh}) \log (p_j m_{jh})$$

$$\log b(p^*) = \log a(p^*) + \beta_0 \prod (p_k m_{kh})^{\beta_k}$$

A nice property of this cost function is that

$$\frac{\partial \log c_h(u,p^*)}{\partial \log p_i^*} = \frac{p_i^* q_i^*}{c_h(u,p^*)} = \frac{p_i q_i}{x_h} = w_{ih}$$

Hence, differentiating (5.11) w.r.t. p_i^*, substituting the indirect utility function in the Hicksian demand functions, we obtain the Marshallian demand functions:

$$(5.13) \quad w_{ih} = \alpha_i + \sum_j \gamma_{ij} \log (p_j m_{jh}) + \beta_i \log (x_h/P_h)$$

where

$$(5.14) \quad \log P_h = \alpha_0 + \sum_k \alpha_k \log (p_k m_{kh}) + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log (p_k m_{kh}) \log (p_j m_{jh})$$

The usual restrictions apply to the parameters α_k, γ_{kj} and β_i. We must also specify the specific equivalence scales m_{ih} and we choose the simple specification

$$(5.15) \quad m_{ih} = 1 + \delta_i a_h$$

where a_h is the number of children below 18 years old in household h, and we normalise m_{ih} to unity for a childless adult couple. No restrictions are implied for the parameters δ_i. This specification is maybe the simplest possible, but it has the advantage of being empirically implementable. It would have been desirable with a specification with the number of children divided with respect to age. One often thinks that older children are more expensive - have greater needs - than younger children, a belief that maybe changed with women's increasing participation rates and hence increasing costs for child care. Anyhow, the definitions of the FES has not allowed us to consistently incorporate different age groups.

Having brought up more than one child, I have the feeling that the first child was the most expensive, i.e. there are economies of scale in children. In (5.15) this effect cannot be tested, but some economies of scale is actually imposed, since $\Delta m_{ih}/m_{ih}=\delta_i/1+\delta_i a_h$. However, in a moment we shall consider another specification which further stresses economies of scale and also allows a test of it.

The general equivalence scale based on AIDS and the Barten cost function, assuming unitary prices, can now be determined as the specification of (3.27):

$$(5.16)\quad c(u_h^0,a_h,1) = c(u_h^0,a_h)/c(u_h^0,1)$$
$$= \exp((\alpha_0 + \sum_k \alpha_k \log(1+\delta_k a_h) + \frac{1}{2}\sum_k\sum_j \gamma_{kj} \log(1+\delta_k a_h)\log(1+\delta_j a_h)) + \prod_k (1+\delta_k a_h)^{\beta_k} (\log x^0 - \alpha_0)) / x^0$$

$\log x^0$ is the total expenditure of a childless couple at reference prices. (5.16) can be used to derive x_N^h as

$$(4.13')\quad x_N^h = x^h/c(u_h^0,a_h,1)$$

Hence, I', the denominator of the price inequality index PI can be derived. For the nominator, I, we also need a cost of living index, as can be seen by the decomposition of x_*^h in (4.13). The appropriate cost of living index is defined by

$$(5.17)\ c(u_h^0, a_h, p^t, p^0) = \exp((\alpha_0 + \sum_k \alpha_k \log p_k^t + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k^t \log p_j^t) + (\log x_N^h - \alpha_0) \prod_k (p_k^t)^{\beta_k}) / x_N^h$$

where the superscript t refer to a particular time period and x_N^h refer to nominal "adult equivalent" total expenditure at the reference utility level. Reference prices are normalised to unity.

Since $x_*^h = x_N^h / c(u_h^0, a_h, p^t, p^0)$, it is now possible to generate the price inequality index PI, through application of a specific inequality measure.

The Barten cost function $c(u, p^*)$ is a specific way of introducing demographic variables in demand analysis, which has attractive intuitive appeal. In this kind of analysis it is the parameters m_{ih} which are defined to depend on the demographic variable a_h.

The Barten-type of introduction of demographic variables has been called <u>demographic scaling</u> by Pollak and Wales(1978,1980). As an alternative to this procedure they propose the method of <u>demographic translation</u>.

According to the latter, there are n translation parameters $(d_1, d_2, \ldots, d_n)$, which translates the n demographic variables $(a_1, a_2, \ldots, a_n)$. Each demand system $(q_i = f_i(x,p))$ can then be replaced by

$$(5.18)\ h_i(p,x) = d_i + \bar{h}_i(p, x - \sum p_k d_k)$$

where only the d's depend on the demographic variables[7]. The translating parameters d_k must also be related to the demographic variables a_h through a specific functional form. Pollak and Wales(1978,1980) considers "linear demographic translating" in particular, and translates the "subsistence" parameters of the linear and quadratic expenditure systems as well as the translog demand system. We shall use a specification which is more general, in which the subsistence as well as the income parameters are translated.

Consider again the AIDS cost function and the following demographic translations:

$$(5.19)\quad \alpha_0 = \alpha_0^* + \varepsilon\, a_h + \theta\, a_h^2$$

$$(5.20)\quad \alpha_i = \alpha_i^* + \phi_i\, a_h$$

$$(5.21)\quad \beta_i = \beta_i^* + \eta_i\, a_h$$

(5.19) and (5.20) incorporates subsistence costs of children and also includes possible economies of scale through the parameter θ. Also the parameter β_i here depends on the number of children and hence take account of differences in the cost of children across incomes.

The cost function now is

$$(5.22)\quad \log c(u_h,p,a_h) = \alpha_0 + \varepsilon\, a_h + \theta\, a_h^2 + \sum_i (\alpha_i^*+\phi_i a_h) \log p_i +$$

$$\frac{1}{2} \sum_i \sum_j \gamma_{ij} \log p_i \log p_j +$$

$$(\beta_i^*+\eta_i a_h)$$

Again using Shephard's lemma, via the Hicksian demand functions, we obtain the Marshallian demand functions

$$(5.23)\ w_{ih} = \alpha_i + \phi_i a_h + \sum_j \gamma_{ij} \log p_j + (\beta_i^* + \eta_i a_h) \log (x_h/P^*)$$

where

$$(5.24)\ \log P^* = \alpha_0 + \varepsilon\, a_h + \theta\, a_h^2 + \sum_i (\alpha_i^* + \phi_i a_h) \log p_i + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k \log p_j$$

The adding-up condition now is

$$(5.25)\ \Sigma\, \alpha_i^* = 1,\ \Sigma\, \beta_i^* = \Sigma\, \eta_i = \sum_i \gamma_{ij} = \Sigma\, \phi_i = 0$$

and the homogeneity and symmetry conditions are like before.

Like in the case with demographic scaling, the significance of the demographic variables, in general, can be tested. However, two additional hypotheses can be tested:

$$(5.26)\ \theta > 0$$

and

$$(5.27)\ \eta_i = 0 \text{ for all } i.$$

The first hypothesis is that there are no economies of scale in children and the second that bliss cost does not depend on the number of children.

Proceeding like before, it is now a straightforward task to get the equivalence scale

$$(5.28)\quad c(u_h^0,a_h,1) = \frac{c(u_h^0,a_h,p^0)}{c(u_h^0,1,p^0)}$$

$$= \exp(\alpha_0 + \varepsilon\, a_h + \theta\, a_h^2 + \sum_i (\alpha_i^* + \phi_i a_h) \log p_i^0 +$$

$$\frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k^0 \log p_j^0 + \prod_k (p_k^0)^{(\eta_k a_h)}$$

$$(\log x^0 - \alpha_0 - \sum \alpha_k \log p_k -$$

$$\sum_k \sum_j \gamma_{kj} \log p_k \log p_j)) \,/\, x^0$$

where p^0 and x^0 refer to reference prices and total expenditure in a reference situation. We now get a new measure of x_N^h corresponding to that in (5.17'), denoted $x_N^h = x^h / c(u_h^0, a_h, 1)$ and we get the new cost of living index as

$$(5.29)\quad c(u_h^0,a_h^0,p^t,p^0) = \exp((\alpha_0 + \sum_k \alpha_k \log p_k^t +$$

$$\frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k^t \log p_j^t) + (\log x_N^h - \alpha_0)$$

$$\prod_k (p_k^t)^{\beta_k}) \,/\, x_N^h$$

with reference prices set to unity. To get the price inequality index is straightforward.

Before turning to the estimation and the empirical results, we should describe the data which are available and the problems associated with these.

5.4 The Data

We shall study the effects of relative-price changes on inequality in Sweden. We will certainly be limited in the time period that is possible to study, since there are considerable limitations as to both the quality and the quantity of data.

Let us start with the generalised AIDS. This demand system has w_{ih} as the dependent variable and hence we need budget shares for different households. These data can only be found in the Family Expenditure Survey. In Sweden the FES has been made three times the last three decades. In addition to budget shares the demand system includes the price vector p, the total expenditure x_h and the number of children a_h. x_h and a_h can be found in the FES. To each FES corresponds one observation of p. Hence, for Swedish data there are only three price observations available during the last three decades! In addition, these three FES are mutually inconsistent w.r.t. the definitions of goods, the concepts of family and income. So, in practice we are actually left with only two price observations.

Without any variation in prices, all the parameters $\delta_1,\dots,\delta_n$ in (5.13) are not possible to identify, as was shown by Muellbauer (1975b).

To remedy this situation there are two options. We can abandon the estimation altogether and trust the official equivalence scales in Sweden. The official sources are actually two and they differ slightly. One source is the Swedish Income Distribution Survey, published by the Central Bureau of Statistics and the other is published by the Social Welfare Board (Socialstyrelsen) as a guide for the local authorities. Since the official equivalence scales are not based on the theory of choice, but rather on physiological or nutritional grounds, this would not be a very satisfactory remedy.

I have instead looked for more price data, i.e. for more family expenditure surveys. I found them in Norway. There, surveys are available from 1967 to 1979 on an annual basis, with the exception of 1968-72. Hence, there are 8 price observations, which is the best we can do. The Norwegian data are also well in accordance with Swedish data; both are done according to the international national accounts

the same.
Since it is not unreasonable to assume that the conditions in Norway and Sweden are not too different, e.g. the attitudes towards children, we shall estimate equivalence scales with Norwegian data and apply these to Sweden.

There are some differences in the design of the FES in Norway in comparison with the Swedish FES. The surveys in 1967 and 1973 are comparable to those in Sweden 1969 and 1978. However, for 1974-76 and for 1977-79 the annual surveys in Norway are done with much smaller samples. In 1973 the sample was 4707 households with a 71 per cent participation rate. In the surveys 1974-76 there were 936 households in 1974, 1117 in 1975 and 1173 in 1976, which is 3226 households and a participation rate of 68 per cent. The Norwegian Central Bureau of Statistics regards the annual figures as less reliable and only publishes averages for the periods 1974-76 and 1977-79. I have used the unpublished annual data to get as many price observations as possible.

In the FES there are 9 commodity groups and for the 8 Norwegian family expenditure surveys there are, all in all, 81 observations. For our demand system, that makes (9-1)81=648 independent observations. For computational reasons we have decreased the number of commodity groups by two. The commodity groups are:

1. FOOD
2. CLOTHING AND FOOTWARE
3. HOUSING, FUEL AND LIGHT
4. FURNITURE AND HOUSEHOLD EQUIPMENT
5. TRANSPORT
6. RECREATION AND EDUCATION
7. MISCELLANEOUS

<u>Food</u> is composed of Bread and cereals, Meat, Fish, Milk and cheese, Eggs, Oils and fats, Vegetables, fruits and berries, Potatoes, Sugar, Sweets and Other food products.

<u>Housing</u> is composed of Captial costs, Insurance, Water charges, Other current expenditures, Rent and hiring of holiday house and Fuel and power.

Furniture is composed of Furniture and carpets, Textiles, Household appliances, Household utensiles and Miscellaneous household goods and services.

Transport is composed of Transport equipment, Operation and maintenance of transport equipment, Public transport services and Postal telephone and telegraph services.

Education is composed of Recreational equipment, Public entertainment and other services, Books, newspapers and periodicals and Education and child care.

Miscellaneous is composed of Personal care, Travel goods, watches and jewellry, Restaurant and hotel services, Financial services, Other services, Medical care and Beverages and tobacco.

The further aggregation done here is that Alcoholic beverages and tobacco and Medical care are contained in Miscellaneous. The aggregation has been done with the implicit price indexes in the Norwegian national accounts. With this aggregation there are (7-1)81=486 observations. The demand system for the Barten cost function has 62 parameters without the homogeneity or symmetry restrictions imposed. Hence, the degrees of freedom is probably not a big problem here.

A child is defined as a person below 16 years old. In the Swedish FES the definition is a bit different: it was below 16 in 1969 but was changed to below 18 years old in 1978. The application of the Norwegian scales on Swedish income data hence assumes that a 17 or 18-year old child costs the same as those under 16.

For the Swedish price and income data, which are used to construct cost-of-living indexes, the commodity groups are approximately the same as in Norway. However, to get time consistent series, the aggregation has been done slightly different. Hence, Alcoholic beverages and tobacco has been aggregated with Food, while Medical care has been dropped altogether. The reason for the latter step is that this commodity group has been redefined occasionally and has been far from homogenous over time. The Swedish time series data are available from 1951-1979 from the national accounts. With seven

55 parameters to estimate with the unconstrained Slutsky matrix based on the Barten cost function.

This concludes the description of data, the sources of which can be found further specified in the list of references. We now turn to the estimations and the empirical results.

6 METHODS OF ESTIMATION AND EMPIRICAL RESULTS

6.1 Introduction

We shall here discuss the stochastic properties and the estimation methods in general. We only refer to the particular specification if it is needed. The estimations are done in two steps, one in which we estimate equivalence scales for Norwegian data, and the other in which we estimate a demand system for Swedish data, in order to get cost of living indexes for different Swedish households. Unless explicitly stated, it is assumed that the stochastic specification and the estimation method are the same in both these estimations.

6.2 Methods of Estimation

To estimate the demand systems above, the equations have to be made stochastic. This is done in the usual way by adding a stochastic term to the right-hand side of each demand equation. Hence, the demand system also consists of the stochastic vector of disturbance terms: $\varepsilon_t=(\varepsilon_{1t},\ldots,\varepsilon_{it},\ldots,\varepsilon_{nt})$.

We shall assume that ε_t has a multivariate normal distribution with $E(\varepsilon_t)=0$ and covariance matrix Ω. Also, the disturbances are assumed to be serially uncorrelated. The assumption of normality is common and can be done with reference to the Central Limit Theorem, since there are probably a number of reasons for the actual demand pattern to deviate from the expected[1)].

Since the equations in the demand system share the same parameter, a single-equation method cannot be used. The test of the symmetry-restriction also requires a system method, i.e. that the covariance matrix Ω is used in the estimation. Since the deterministic parts of the equations in the system identically add to unity, Ω is singular. This problem can here be solved by deleting one equation in the estimation. We used Ω_0 instead, without Miscellaneous; i.e. the last row and column of Ω deleted.

Since there are only seven commodities in the system and we have (7-1) 81 = 486 independent observations, it is possible to use Ω_O, i.e. to estimate all the covariances, in the estimation, which is what we have done here.

The equivalence scales obtained from the Norwegian data are used to deflate x^h, the nominal total expenditure in the Swedish family expenditure surveys in 1969 and 1978. Then we arrive at the nominal total expenditures of adult couple equivalent households, x_N^h, for which we compute inequality indexes.

To obtain the inequality indexes defined for x_*^h, however, we must calculate the cost of living index in (4.3). This is done by using Swedish time series data from the national accounts. The same commodity groups were used and the estimation was done for the period 1963-1979, with annual data.

A maximum likelihood method is used to estimate the demand systems. When possible, multivariate regression was applied, but in one case, when the Barten cost function was used, we used full information maximum likelihood (FIML). In the latter case it was very important to have good starting values for the parameters.

6.3 The Estimated Demand Systems

In this section we report the estimated demand systems for Norway and Sweden. In the next chapter we give the estimated equivalence scales and cost of living index numbers.

In Table 6.1 we report the symmetry-constrained estimation of the demand system based on Barten's cost function. In Tables 6.2 - 6.4 we present the estimated demand systems based on demographic translation. In the latter case we also test the economic theoretical restrictions, homogeneity and symmetry. This was not done in the first case, since the computational burden inherent in the nonlinear Barten model was so high. In addition, the primary concern here is not the testing of the neoclassical theory of consumption.

The estimation of the demand system based on Barten's cost function was not very easy with the nonlinearities involved. It turned out that good starting values for the parameters were essential for convergence, particularly for the γ_{ij}:s and the δ_i:s. By experimenting with different plausible values on the δ_i-parameters, convergence was finally achieved[2)] and the resulting estimates are presented in Table 6.1.

The estimates seem to be fairly plausible, though the δ_i-parameters seem to be a little bit too small[3)]. For instance, the δ_1 parameter implies that a child, on average, needs 17 per cent of the food consumption of a married couple; or, equivalently, the arrival of a child adds about 17 per cent to food cost for a childless adult couple. The parameter for Clothing is the highest, 0.60, and it is lowest for Miscellaneous, -0.09. The negative parameters implies that a family with children need less of that particular good compared to childless families. According to the estimates, this is so for Furniture, Transport and Miscellaneous.

Looking further into the estimates we see that Food and Housing are classified as necessities. The budget shares for poor and rich households are implied by the α_i- and β_i-parameters.

In Table 6.2 the budget shares for poor and rich households are reported. The largest differences are for the budget shares of Food and Transport. The subsistence household allocates 48 per cent to Food and 2 per cent to Transport. The rich household's figures are 33 and 13, respectively. In general, these estimates seem quite plausible.

	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	β_i	δ_i	R^2	D-W
	.4810	-.1452	-.1300	.0629	.1114	-.0968	.3362	-.1386	-.1508	.1749	.949	1.283
	(61.56)	(-1.01)	(-1.89)	(.30)	(.96)	(-.59)	(4.52)	(-.88)	(-27.90)	(.76)		
ING	.0760	-.1300	.0635	.0006	.1383	-.0656	.0069	-.0138	.0085	.6049	.595	2.170
	(13.77)	(-1.89)	(.70)	(.003)	(2.05)	(-.56)	(.10)	(-.17)	(2.94)	(1.38)		
NG	.1914	.0629	.0006	.4135	-.3630	-.0539	-.2588	.1987	-.0248	.0983	.594	1.835
	(15.88)	(.30)	(.003)	(.67)	(-1.21)	(-.11)	(-1.52)	(.55)	(-4.33)	(.51)		
TURE	.0810	.1114	.1383	-.3630	.1813	.1166	-.1821	-.0025	.0113	-.0160	.292	1.685
	(8.70)	(.96)	(2.05)	(-1.21)	(.91)	(.42)	(-2.01)	(-.01)	(2.19)	(-.09)		
PORT	.0188	-.0968	-.0656	-.0539	.1166	.2448	.1630	-.3079	.1143	-.0630	.839	1.653
	(1.32)	(-.59)	(-.56)	(-.11)	(.42)	(.59)	(1.08)	(-.95)	(14.08)	(-.58)		
TION	.0581	.3362	.0069	-.2588	-.1821	.1630	-.0961	.0309	.0154	.1737	.339	1.574
	(7.56)	(4.52)	(.10)	(-1.52)	(-2.01)	(1.09)	(-1.08)	(.31)	(3.75)	(.79)		
LLANEOUS	.0937	-.1386	-.0138	.1987	-.0025	-.3079	.0309	.2332	.0262	-.0939	.462	1.648
	(7.04)	(-.88)	(-.17)	(.55)	(-.01)	(-.95)	(.31)	(.60)	(4.89)	(-.87)		

6.1. Results from maximum likelihood estimation of demand system with Barten cost function.

$\alpha_0 = 8.987$.

Log of likelihood function = 1423.79.

t-values are in parentheses.

Table 6.2. Budget shares for "subsistence" and "bliss" households in Norway, implied by the parameters α_0 and β_0.

Commodity	Poor	Rich
Food	.4810	.3302
Clothing	.0760	.0845
Housing	.1914	.1666
Furniture	.0810	.0923
Transport	.0188	.1331
Education	.0581	.0735
Miscellaneous	.0937	.1199

As can be seen in Table 6.1, the δ_i-parameters in general are insignificant. To test the overall significance of the Barten cost function - the significance of the general equivalence scale - we estimated the system again, with all δ_i-parameters restricted to zero. Since the sample is fairly large here, we can apply the likelihood ratio test. The meaning of the LR-test is that twice the difference between the logarithmic likelihood value for the model with n free parameters and the corresponding value for the more restricted model with n-k free parameters is asymptotically chi-square distributed with k degrees of freedom under the null hypothesis of the more restricted model.

Hence, restricting the model (5.13) by setting all δ_i equal to zero, implies a chi-squared likelihood ratio with 7 degrees of freedom. The log likelihood value of the restricted model is 1367.44 against 1423.79 for the Barten hypothesis in (5.13). The critical value at the 1% significance level is 18.48. Hence, the hypothesis that the number of children does not affect the consumption pattern, or, that the cost of children is zero, is clearly rejected by our data.

Before turning to the estimates of equivalence scales and cost of living index numbers, we shall consider the alternative estimates based on the method of demographic translation, as suggested by Pollak and Wales (1978,1980).

Consider again the demographic translation implied by (5.19) - (5.21) and the translated demand system (5.23). Consider also the homogeneity and symmetry restrictions in (5.9). The estimates from these three demand systems are given in Tables 6.3 - 6.5.

	α_i^*	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	β_i^*	ϕ_i	η_i	R^2	D
FOOD	.4519	1.3556	-2.1174	.0313	-.9195	3.1203	-1.9746	-.2599	-.1582	-.0119	.0057	.946	1.
	(33.34)	(1.100)	(-1.34)	(.07)	(-1.09)	(1.83)	(-2.75)	(-.55)	(-27.05)	(-2.01)	(1.35)		
CLOTHING	.0756	-.8692	.1471	.0742	.4610	-.5652	.4693	.4023	.0101	.0102	-.0003	.627	2.
	(10.38)	(-1.31)	(.17)	(.32)	(1.01)	(-.62)	(1.22)	(1.59)	(3.41)	(3.19)	(-.14)		
HOUSING	.2049	1.0939	.5309	.5862	-.8938	-.4386	-.7508	-.1623	-.0348	-.0233	.0130	.645	2.
	(15.87)	(.93)	(.35)	(1.43)	(-1.11)	(-.27)	(-1.10)	(-.36)	(-6.65)	(-4.12)	(3.22)		
FURNITURE	.0933	.3113	.3566	.1939	-.0314	-.5461	.0087	-.2373	.0145	-.0009	-.0007	.242	1.
	(9.56)	(.35)	(.31)	(.63)	(-.05)	(-.44)	(.02)	(-.70)	(3.67)	(-.23)	(-.24)		
TRANSPORT	.0148	-1.4285	.0650	-.6994	1.1064	-.1735	1.4108	.0170	.1190	.0154	-.0089	.861	1.
	(.93)	(-.98)	(.04)	(-1.38)	(1.11)	(-.09)	(1.66)	(.03)	(18.30)	(2.20)	(-1.79)		
EDUCATION	.0690	-.5294	1.5851	-.5454	.2372	-1.6211	.8329	.4518	.0144	.0008	.0027	.489	1.
	(6.33)	(-.53)	(1.25)	(-1.58)	(.35)	(-1.18)	(1.44)	(1.19)	(3.16)	(.16)	(.78)		
MISCELLANEOUS	.0905	.0663	-.5673	.3592	.0401	.2241	.0038	-.2115	.0350	.0099	-.0114	.571	2.
	(9.18)	(.07)	(-.50)	(1.16)	(.07)	(.18)	(.01)	(-.59)	(8.80)	(2.30)	(-3.72)		

Table 6.3. Results from multivariate regression estimation of demand system with demographic translation.

Fixed parameters: α_0 = 8.987, ε = 0.2357 and θ = -0.019

Log of likelihood function = 1385.80

t-values are in parentheses.

	α_i^*	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	β_i^*	ϕ_i	η_i	R^2
FOOD	.4550	1.7772	1.7534	.1881	-1.5999	-1.7613	-.6976	.3402	-.1582	-.0109	.0050	.938
	(31.58)	(1.36)	(1.57)	(.41)	(-1.83)	(-1.95)	(-1.08)	(.73)	(-27.05)	(-1.73)	(1.11)	
CLOTHING	.0751	-.9364	-.4592	.0493	.5682	.1992	.2701	.3088	.0101	.0100	-.0002	.622
	(10.29)	(-1.42)	(-.81)	(.21)	(1.28)	(.44)	(.82)	(1.79)	(3.41)	(3.13)	(-.09)	
HOUSING	.2052	1.1277	.7178	.5923	-.9298	-.6643	-.7004	-.1433	-.0348	-.0233	.0130	.645
	(15.92)	(.97)	(.72)	(1.46)	(-1.19)	(-.82)	(-1.21)	(-.70)	(6.65)	(4.12)	(3.22)	
FURNITURE	.0930	.2742	.0702	.1823	.0207	-.1883	-.0816	-.2775	.0145	-.0010	-.0007	.241
	(9.55)	(.31)	(.09)	(.59)	(.04)	(-.31)	(-.19)	(-1.30)	(3.67)	(-.24)	(-.22)	
TRANSPORT	.0137	-1.5747	-1.4302	-.7602	1.3634	1.7214	.9050	-.2247	.1190	.0150	-.0086	.859
	(.86)	(-1.08)	(-1.15)	(-1.50)	(1.40)	(1.71)	(1.25)	(-.17)	(18.30)	(2.13)	(-1.71)	
EDUCATION	.0672	-.7737	-.5285	-.6251	.6138	1.0202	.1682	.1251	.0144	.0002	.0030	.458
	(5.99)	(-.76)	(-.61)	(-1.77)	(.90)	(1.45)	(.33)	(.31)	(3.16)	(.04)	(.85)	
MISCELLANEOUS	.0907	.1057	-.1235	.3733	-.0364	-.3269	.1363	-.1286	.0349	.0101	-.0115	.569
	(9.24)	(.36)	(-.64)	(1.22)	(-.07)	(-1.09)	(.47)	(-.47)	(8.77)	(2.34)	(-3.74)	

Table 6.4. Results from multivariate regression estimation of demand system with demographic translation.
The system is homogeneity-constrained.
Fixed parameters: α_0 = 8.987, ε = 0.2357 and θ = -0.019.
Log of likelihood function = 1377.93.

	α_i^*	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	β_i^*	ϕ_i	η_i	R^2	D-W
OD	.4731	-.4807	-.1401	-.0044	.1262	-.1769	.2596	.4164	-.1590	-.0125	.0057	.932	1.115
	(45.70)	(-.91)	(-.32)	(-.02)	(.37)	(-.50)	(1.01)	(1.84)	(-25.95)	(-1.90)	(1.20)		
OTHING	.0688	-.1401	.1513	.0169	.0298	-.2551	-.0443	.2416	.0102	.0104	-.0004	.595	2.191
	(11.40)	(-.32)	(.40)	(.10)	(.10)	(-.86)	(-.20)	(1.48)	(3.32)	(3.14)	(-.18)		
USING	.2054	-.0044	.0169	.4077	-.2796	-.0227	-.2986	.1808	-.0344	-.0231	.0130	.626	1.941
	(21.57)	(-.02)	(.10)	(1.27)	(-1.56)	(-.09)	(-2.62)	(.84)	(-6.41)	(-3.98)	(3.14)		
RNITURE	.0883	.1262	.0298	-.2796	.2237	.1788	-.0702	-.2088	.0143	-.0011	-.0008	.214	1.701
	(12.34)	(.37)	(.10)	(-1.56)	(.88)	(.65)	(-.41)	(-1.29)	(3.57)	(-.25)	(-.26)		
ANSPORT	.0128	-.1769	-.2551	-.0227	.1788	.3880	.2974	-.4094	.1197	.0159	-.0087	.851	1.722
	(1.16)	(-.50)	(-.86)	(-.09)	(.65)	(1.14)	(1.66)	(-1.93)	(17.93)	(2.21)	(-1.69)		
UCATION	.0649	.2596	-.0443	-.2986	-.0702	.2974	-.0681	-.0758	.0141	.0001	.0029	.371	1.690
	(8.46)	(1.01)	(-.20)	(-2.62)	(-.41)	(1.66)	(-.50)	(-.70)	(2.86)	(.03)	(.76)		
SCELLANEOUS	.0868	.4164	.2416	.1808	-.2088	-.4094	-.0758	-.1447	.0350	.0102	-.0116	.564	1.786
	(10.07)	(1.84)	(1.48)	(.84)	(-1.29)	(-1.93)	(-.70)	(-.52)	(8.73)	(2.36)	(-3.74)		

ble 6.5. Results from multivariate regression estimation of demand system with demographic translation.
The system is symmetry-constrained.
Fixed parameters: $\alpha_0 = 8.987$, $\varepsilon = 0.2357$ and $\theta = -0.019$
Log of likelihood function = 1363.47

In these three cases multivariate regression was computationally less burdensome and was therefore used instead of FIML. Note therefore that the log of the likelihood values are not comparable to the FIML values obtained in the previous estimation. However, the LR test still applies[4).

Like before the estimation of α_0^* turned out to be problematic. Since $\alpha_0^* = \alpha_0 + \varepsilon a_h + \theta a_h$, we guessed some values for ε and θ. α_0 was fixed at 8.987 as before and $\varepsilon=0.2357$ and $\theta=-0.019$. Since α_0^* is the subsistence total expenditure for a certain household it was reasonable to use the equivalence scales used in the Swedish social assistence allowance. In these scales a single adult is the reference household. An adult couple is 1.8 and a child on average 0.45 adult units. The chosen values for ε and θ approximates this official Swedish scale, hence also recognising the economies of scale in children.

Again, it is interesting to test for the overall significance of the demographic translation. We did that through the LR-test for the symmetry-constrained translated demand system and the restricted model given by (5.7). The log of the likelihood function was 1363.47 and 1305.17 for the restricted model. Hence, the chi-squared LR of 116.6 against the critical value at the 1% level, with 14 degrees of freedom, of 29.14 implies a rejection of the restricted model. But not only is the restricted model rejected here, but very interesting results follow from the test of homogeneity and symmetry, which can be seen in Tables 6.3 - 6.5.

The test of homogeneity can be done equation by equation through a t-test on $\Sigma \gamma_{ij}$. Here we have, however, tested homogeneity on the whole system by imposing it in the system. The log likelihood value of the homogeneity-restricted system is 1377.93 against the value of the unrestricted system, 1385.80. Twice the difference is 15.74 against the critical value at the 1% level, 16.81. Hence, homogeneity is accepted at the 1% significance level. However, homogeneity is rejected at the 2.5% level[5).

In testing Slutsky symmetry one is bound to use the large sample LR-test, since this theoretical restriction goes across the equations. In Table 6.5 the symmetry-constrained estimates are given.

The test of symmetry can actually be done in two ways. It can be done _given_ homogeneity or, alternatively, _given_ the unconstrained system. Mizon (1977) is of the opinion that symmetry only can be tested if homogeneity is _not_ rejected. However, according to Deaton and Muellbauer (1980), this is only true if the unconstrained system unambiguously can be treated as a maintained hypothesis. Since many economists would regard a homogeneity-constrained system as maintained hypothesis, the way of testing symmetry is somewhat a matter of choice. The latter test seem to be somewhat easier to pass, so that symmetry not has been rejected as often as homogeneity in previous empirical studies[6].

The unconstrained system has 42 free γ_{ij}-parameters against the 36 in the homogeneity-constrained system. In the symmetric system there are only 27 free parameters. The log of the likelihood function is 1363.47 for the symmetric system. The double log likelihood values and the critical values at the 1% significance level of the chi-square distribution are given in Table 6.6.

Table 6.6. Log likelihood values for different demand models and corresponding critical values of the χ^2-distribution.

System	2 log L	Differences	
Unrestricted	2771.60	-	-
Homogenous	2755.86	15.74	-
Symmetric	2726.94	28.92	44.66

Critical values of the chi-square distribution

	1 %	5 %
$2\ \chi^2(6)$	16.81	12.59
$2\ \chi^2(15)$	30.58	25.00
$2\ \chi^2(21)$	38.65	32.41

Making the test at the 1% level and treating the homogenous system as the maintained hypothesis, we see that symmetry cannot be rejected (LR=28.92 against the critical value $\chi^2(15)=30.58$). However, like for homogeneity, symmetry is rejected at the 5% level.

Since θ was fixed in the estimation, it was not possible to test for economies of scale in children. However, the hypothesis that $\eta_i=0$, for all i, could be tested. This hypothesis says that the number of children does not affect the classification of commodities in luxuries and necessities. Actually, one would expect that children need relatively more of necessities and hence that $\eta_i>0$ for necessities and $\eta_i<0$ for luxuries. This is also confirmed by the estimates in tables 6.3 - 6.5. η_i is significantly positive for Housing and significantly negative for Miscellaneous.

The estimated demand systems for Norwegian data, presented so far in this chapter, had the sole purpose of being done to estimate general equivalence scales. These general equivalence scales are then used to adjust the nominal expenditures in the Swedish family expenditure surveys of 1969 and 1978 into a needs-corrected basis. The expenditures so arrived at are then applied to parameter estimates obtained for Swedish time series for the period 1963-1979[7)]. These estimates are obtained on the presumption that the aggregate index of demographic characteristics, k in (5.7''), is constant <u>or</u> that deviations in it are independently distributed from those in the average budget share $\bar{w}$. In these cases no bias occurs in the estimated parameters, but merely the constants α_i are changed to $\bar{\alpha}_i=\alpha_i-\beta_i\log\bar{k}$, where $\bar{k}$ is the sample mean value of k.

Again fixing α_0 at the lowest observed total expenditure in the family expenditure survey 1978, and deflating back to 1975, the year when prices were normalised to unity, the demand system was estimated on aggregate price and income data from the Swedish national accounts.

	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	β_i	R^2
FOOD	0.315	-0.006	0.163	-0.038	-0.145	-0.080	0.116	0.020	-0.044	0.994
	(20.50)	(-0.23)	(2.32)	(-1.23)	(-3.02)	(-2.27)	(4.12)	(0.89)	(-2.68)	
CLOTHING	0.149	-0.054	-0.041	-0.004	0.122	-0.007	-0.060	-0.071	-0.071	0.969
	(10.90)	(-2.35)	(-0.65)	(-0.14)	(2.89)	(-0.21)	(-2.41)	(0.79)	(-4.85)	
HOUSING	0.294	0.137	-0.079	0.011	-0.131	0.144	0.107	-0.088	-0.070	0.967
	(12.13)	(3.34)	(-0.72)	(0.22)	(-1.72)	(2.61)	(2.45)	(-2.50)	(-2.68)	
FURNITURE	0.051	-0.015	-0.014	0.040	0.068	-0.029	-0.091	0.001	0.034	0.839
	(5.33)	(-0.93)	(-0.31)	(2.03)	(2.23)	(-1.33)	(-5.19)	(0.11)	(3.23)	
TRANSPORT	0.056	-0.032	-0.027	0.030	0.062	-0.117	-0.005	0.037	0.094	0.925
	(3.72)	(-1.29)	(-0.04)	(1.02)	(1.35)	(-3.42)	(-0.20)	(1.71)	(5.87)	
EDUCATION	0.037	0.033	0.037	0.027	0.023	0.094	-0.057	-0.073	0.077	0.992
	(3.57)	(1.92)	(0.77)	(-1.28)	(0.73)	(3.96)	(-2.99)	(-4.82)	(6.91)	
MISCELLANEOUS	0.098	-0.063	-0.038	-0.012	0.001	-0.004	-0.011	0.087	-0.019	0.930
	(10.38)	(-3.98)	(-0.89)	(-0.01)	(0.02)	(-0.18)	(-0.61)	(6.24)	(-1.86)	

Table 6.7. Results from multivariate regression estimation of demand system for Swedish time series data 1963-1979.

Fixed parameters: $\alpha_0 = 9.903$. Log of likelihood function = 1175.79

t-values are in parentheses.

Multivariate regression was used and the estimated parameters for the unrestricted system are presented in Table 6.7. Again, the R^2's are the square of the simple correlation coefficient between w_i and $\hat{w}_i$. As before, the α_i- and β_i-parameters are highly significant. In this case there are also more price parameters, γ_{ij}, which are significant; 20 out of 49 price parameters are significant at the 5% significance level of the t distribution.

The overall fit of the equations are quite good, especially when considering that it is budget shares which are the dependent variables. A difference compared to the previous estimates on Norwegian micro data, is that Clothing is significantly classified as a necessity, while Miscellaneous is also classified as a necessity, but is not significant at the 5% level.

In Table 6.8 the tests of homogeneity and symmetry are presented. As can be seen, both homogeneity and symmetry are rejected, hence conforming well with previous empirical results for other countries (cf. Barten (1969) or Deaton and Muellbauer (1980)). It is the estimated parameters in Table 6.7 which are used to compute the cost of living indexes for different utility levels. Since the dependent variables are budget shares, the estimated parameters are not the elasticities. To get an idea of how sensitive the demand is to changes in prices and income, we have computed the income and the compensated and uncompensated price elasticities for the unrestricted, homogenous and symmetric demand systems. The reported elasticities are <u>means</u> for the sample period.

Table 6.8. Log likelihood values and likelihood ratio tests. The number of restrictions are given in parentheses.

Model	2 log L	Difference
Unrestricted	1175.79	-
Homogenous	1149.04	26.75
Symmetric	1062.53	86.51

Critical values of the χ^2-distribution.

	5 %
$\chi^2(7)$	14.07
$\chi^2(21)$	32.67

	Unrestricted			Homogenous			Symmetric		
	e_{ix}	s_{ii}	$\bar{s}_{ii}$	e_{ix}	s_{ii}	$\bar{s}_{ii}$	e_{ix}	s_{ii}	$\bar{s}_{ii}$
FOOD	0.85	-0.78	-1.07	0.84	-0.81	-1.04	0.80	-0.94	-1.16
CLOTHING	0.17	-1.59	-1.60	0.20	-0.89	-0.91	-0.25	-2.97	-2.95
HOUSING	0.69	-0.77	-1.05	0.65	-1.23	-1.38	1.22	-0.41	-0.69
FURNITURE	1.42	-0.02	-0.13	1.47	-0.69	-0.81	0.85	-1.50	-1.57
TRANSPORT	1.69	-1.59	-1.83	1.72	-1.64	-1.89	1.51	-1.50	-1.57
EDUCATION	1.83	-1.38	-1.58	1.80	-1.49	-1.68	1.99	-1.40	-1.61
MISCELLANEOUS	0.78	0.07	0.00	0.82	-0.23	-0.30	0.62	-0.84	-0.89

Table 6.9. Income and compensated and uncompensated own-price elasticities, for Swedish time-series data 1963-1979.

s_{ii} is the compensated own-price elasticity, $\bar{s}_{ii}$ the uncompensated own-price elasticity and e_{ix} the income elasticity.

The elasticities have been computed for the unrestricted, homogenous and symmetric demand systems, respectively.

The general impression of these elasticities is that they are quite high, but are in conformation with expectations based on economic theory. All compensated own-price elasticities are negative, except in one case, for Miscellaneous in the unrestricted system. There are no inferior goods, except Clothing in the symmetric system, which is what one would expect in a study with aggregate commodities. Note also that the estimated elasticities are quite sensitive to the particular specification used, i.e. whether it is unrestricted, homogenous or symmetric.

To give an idea of any possible differences between the Norwegian and the Swedish estimates, we can compare the budget shares for the "subsistence" and "bliss" households. These budget shares for the Swedish data are reported in Table 6.10 and refer to 1975, the year with unity-normalised prices.

As can be seen, there are some differences in these estimates. For instance, the budget share for Food for a poor household is much larger in Norway. On the other hand, Transport, which is a luxury, has a smaller budget share for the poor household in Norway. These facts probably reflects the relatively lower standard of living in Norway in the middle of the 1970s. On the other hand, the differences may be a bit exaggerated, since it may be that α_0 has been chosen relatively lower in Norway.

When computing the equivalence scales for the Swedish households, these differences are taken account of, since the scales are directly applied to Swedish total expenditures.

Table 6.10. Budget shares for "subsistence" and "bliss" households in Sweden 1975, implied by the parameters α_0 and β_0.

Commodity	Poor	Rich
Food	.315	.271
Clothing	.149	.078
Housing	.294	.224
Furniture	.051	.085
Transport	.056	.150
Education	.037	.114
Miscellaneous	.098	.079

To get a general idea of the performance of these estimates, we can compare them to the extensive experiments with different specifications done in Klevmarken (1981). He compared the constant elasticity of demand system, the linear expenditure system (with and without habit formation), the Rotterdam system and the translog system (based on the indirect utility function)[8].

Klevmarken's estimations were done for three different data sets for the period 1950-1970. Food was divided into two commodity sets, with 4 and 8 items, respectively, and all the commodities were grouped into 4 items. The latter were Food, beverages and tobacco, Housing services, Clothing and Other goods and services.

For the 8-commodity breakdown, in no case were all the compensated own-price elasticities negative. In the Rotterdam model half of these elasticities were positive! For the other two commodity breakdowns the linear expenditure system with habit formation performed best, with all compensated own-price elasticities being negative and all income elasticities positive.

Looking at the goodness of fit, this was quite high for the two 4-commodity breakdowns. An exception was the Rotterdam model, which showed low, and sometimes negative, R^2's for all the commodity breakdowns. In general, the goodness of fit was better for the more aggregated data. For the 8-commodity breakdown the translog model performed well.

It should here be stressed that we cannot draw any firm conclusions from these comparisons, since they are done for data from different time periods. Anyhow, I think that the results obtained here are quite satisfactory and points to the importance of the underlying model.

With these basic estimates we turn to the estimation of equivalence scales and cost of living index numbers, both of which are based on these estimated demand systems.

7 THE ESTIMATED EQUIVALENCE SCALES

7.1 Introduction

The construction of equivalence scales has been interesting social scientists for a long time. For instance, in conducting social policy there has been a need for equivalence scales in designing social assistence allowances.

The theoretical foundation for the equivalence scales in this study has been analysed in chapter 3. There are, however, at least three other methods which can be used for this purpose[1]. The first of these methods is maybe the most simple one and is most frequently used in the official Swedish social policy: discretionary estimates based on approximate apprehensions of the basic needs of adults and children.

The second approach is based on measures of nutritional needs. These scales could simply be based on physiological studies, where the number of calories needed for men, women and children in different ages are measured. This method was used in the so called Amsterdam scale, computed by the statistical bureau in Amsterdam in the beginning of this century[2].

A third approach has been suggested by Kapteyn and van Praag (1976). They used a survey method where a large number of individuals were asked to evaluate their incomes, x, and these individuals' welfare levels, μ, were used in a regression on family size and income. The equivalence scales obtained then tries to answer the question: what compensation, in terms of x, is needed for a family of size 1, to be as well off, in terms of μ, as a family of size 0[3].

In none of these studies are the scales based on a proper economic theory. For instance, the fact that the cost of children may differ across households with different incomes are not always accounted for.

In the sequel we present estimates of equivalence scales for Norwegian data, which are based on the neoclassical theory in the previous chapters. Our estimates are also compared to previous scales based on other methods.

7.2 The Estimated Equivalence Scales

In tables 7.1 and 7.2 we present the estimated equivalence scales for the Norwegian data. In Table 7.1 we give the equivalence scales based on Barten's cost function and in Table 7.2 the equivalence scales based on the method of demographic translation.

As can immediately be seen, the former scales are lower than the latter. One should then keep in mind that it is only in the former case that all demographic parameters are kept free in the estimation. In the demographically translated demand system, the parameters ε and θ, in the "subsistence" cost function log a(p), was fixed and the values was chosen from the official Swedish scales. In the latter case, therefore, the estimates are biased towards those scales.

Another noteworthy difference between the two sets of estimates is that the equivalence scales based on Barten's cost function are much more sensitive to changes in total expenditure (income). For instance, the scale is 1.321 for a 30000 Ncr household with 4 children against 1.146 for the 110000 Ncr household. For the demographically translated system the scales are 1.891 and 1.884, respectively. In the first case the difference is 17 percentage points against 0.7 in the latter case.

Table 7.1. Equivalence scales based on Barten's cost function. For Norwegian data, in 1974 year's prices. An adult childless couple is normalised to unity.

Total expenditure : No. of children	1	2	3	4
20000	1.111	1.210	1.299	1.381
30000	1.098	1.182	1.256	1.321
40000	1.088	1.162	1.226	1.280
50000	1.081	1.147	1.203	1.249
60000	1.075	1.135	1.185	1.224
80000	1.066	1.117	1.156	1.186
110000	1.056	1.096	1.126	1.146
140000	1.048	1.081	1.103	1.116

Table 7.2. Equivalence scales based on demographic translation of the demand system. For Norwegian data, in 1974 year's prices. An adult childless couple is normalised to unity.

Total expenditure	No. of children 1	2	3	4
20000	1.243	1.485	1.708	1.892
30000	1.242	1.484	1.707	1.891
40000	1.241	1.483	1.706	1.890
50000	1.241	1.483	1.705	1.888
60000	1.241	1.482	1.704	1.887
80000	1.240	1.481	1.703	1.885
110000	1.240	1.481	1.702	1.884
140000	1.240	1.480	1.701	1.882

In Table 7.3 the estimated scales are compared to two official Swedish equivalence scales. One of these is from the Swedish income distribution survey and the other from the Social Board, the latter which is particularly designed for social assistence allowances in the municipalities. Certainly, it can be seen here that the equivalence scales based on demographic translation are close to those in the Swedish income distribution survey. The official scales are, however, above both our estimated scales for all total expenditure levels. It is also clear from the figures that the economies of scale are most significant for the estimates based on Barten's cost function, but less so for the other scales. For the Barten scale the first child increases cost with 9.8 per cent against 5.2 per cent for the lowest expenditure. This could be compared to the official scales in the Swedish income distribution survey, which are 24.2 and 14.1, respectively. This effect is more pronounced when total expenditure rise. For the household with 110000 Nkr total expenditure the corresponding figures, for the Barten scale, are 5.6 and 1.8 per cent, respectively.

Table 7.3. Comparison of different equivalence scales. An adult childless couple is normalised to unity.

No. of children	Equivalence scale Income	Barten	Demographic translation	Sw inc. distr. surv.	Social ass. allowance
1	30000	1.098	1.242	1.242	1.250
2	30000	1.182	1.484	1.485	1.500
3	30000	1.256	1.707	1.727	1.750
4	30000	1.321	1.891	1.970	2.000
1	50000	1.081	1.241	1.242	1.250
2	50000	1.147	1.483	1.485	1.500
3	50000	1.203	1.705	1.727	1.750
4	50000	1.249	1.888	1.970	2.000
1	60000	1.075	1.241	1.242	1.250
2	60000	1.135	1.482	1.485	1.500
3	60000	1.185	1.704	1.727	1.750
4	60000	1.224	1.887	1.970	2.000
1	80000	1.066	1.240	1.242	1.250
2	80000	1.117	1.481	1.485	1.500
3	80000	1.156	1.703	1.727	1.750
4	80000	1.186	1.885	1.970	2.000
1	110000	1.056	1.240	1.242	1.250
2	110000	1.096	1.481	1.485	1.500
3	110000	1.126	1.702	1.727	1.750
4	110000	1.146	1.884	1.970	2.000

So far we have given the relative changes in costs due to different number of children, as given by equivalence scales. However, it is instructive to study also the differences, measured in the monetary unit. The measure which corresponds to the cost of living index is the compensating variation, which can also be defined for different demographic profiles. In the latter case we get

$$(7.1)\quad CV = c(u^0,p^0,a^h) - c(u^0,p^0,a^0)\ ,$$

which simply is the difference instead of the relative change as in (3.26). In Table 7.4 we give the compensating variation, CV, for the Barten cost function and for the demographically translated cost function. CV is column (2) minus column (1) for the former and column (4) minus column (1) for the latter. Columns (3) and (5) show the increase in cost for an additional child, up to four, for different expenditure levels.

Table 7.4. The compensating variation based on Barten's cost function and demographic translation. For Norwegian data, in 1974 year's prices.

	(1)	(2)	(3)	(4)	(5)
No. of children	Expenditure for reference household	Barten	Diff	Demographic translation	Diff
1	30000	32940	2940	37260	7260
2	30000	35460	2520	44520	7260
3	30000	37680	2220	51210	6690
4	30000	39630	1950	56730	5520
1	50000	54050	4050	62050	12050
2	50000	57350	3300	74150	12100
3	50000	60150	2800	85250	11100
4	50000	62450	2300	94400	9150
1	60000	64500	4500	74460	14460
2	60000	68100	3600	88920	14460
3	60000	71100	3000	102240	13320
4	60000	73440	2340	113220	10980
1	80000	85280	5280	99200	19200
2	80000	89360	4080	118480	19280
3	80000	92480	3120	136240	17760
4	80000	94880	2400	150800	14560
1	110000	116160	6160	136400	26400
2	110000	120560	4400	162910	26510
3	110000	123860	3300	187220	24310
4	110000	126060	2200	207240	20020

The economies of scale are obvious from columns (3) and (5). Looking at first to the results for Barten's cost function, for a relatively poor household, the annual cost for the first child is 2940 Nkr against 1950 Nkr for the fourth child, which is 66 % of the cost of the first child. For the relatively rich household the cost for the first child is 6160 Nkr against 2200 Nkr for the fourth child, which is only 36 % of the cost of the first child.

As can be seen in columns (4) and (5), the results are quite different for the demographically translated cost function. Here, the economies of scale in children are independent of income, and the cost of children is generally higher. The cost of the fourth child is here 76 % of the first, which seems to be a very high figure.

Since the estimates of the two different equivalence scales are so different, it is worth further consideration of the reasonableness of these scales. We shall do that by considering the <u>specific</u> equivalence scales in the Barten cost function and by comparing our two sets of estimates with estimates for other countries.

Consider then the interpretation of these specific scales from the Barten direct utility function: $u=u(q_1^*,q_2^*,\dots,q_n^*)$ where $q_i^*=q_i/m_{ih}$ and $m_{ih}=1+\delta_i a_h$ is the specific equivalence scale for commodity i. m_{ih} reflects the difference in needs for families with different numbers of children.

Table 7.5. Specific equivalence scales based on estimation with Barten's cost function. A childless adult couple is normalised to unity.

Commodity	No. of children	1	2	3	4
Food		1.175	1.350	1.525	1.700
Clothing		1.605	2.210	2.815	3.420
Housing		1.098	1.197	1.295	1.393
Furniture		.984	.968	.952	.936
Transport		.937	.874	.811	.748
Education		1.174	1.347	1.521	1.695
Miscellaneous		.906	.812	.718	.624

From the specific scales in Table 7.5 we see that the need will increase for four of the goods, but decrease for three of the goods. The decrease is for Furniture, Transport and Miscellaneous. For the last two goods this result might be reasonable. It is not unreasonable that households with children are less mobile and that they need less of Alcohol and tobacco, Financial services and Restaurant and hotel services. However, for Furniture and household equipment the result seems to be a bit odd. A priori, one would like to think that a larger household should need more furnitures and household appliances as well as services.

A child needs, on average, 18% of the food need for a childless couple and 60% of the clothing need. For Housing the additional need is 10% and for Education it is 17%. To me, these estimates seem reasonable. For instance, the Housing estimate seem to be a rather accurate estimate of the marginal cost of a fourth room. The estimate for Clothing might seem a little bit high, but, taken together, these scales are fairly close to the estimates in Seneca and Taussig (1971) for Necessities.

Finally, we can compare our estimates to those for other countries. Here we have chosen some different estimates for U.K., U.S.A. and Holland. Some of these studies have been summarised by Nicholson (1976).

Our estimates, based on Barten's cost function, are close to those found by Kapteyn and van Praag (1976), Nicholson (1976) and Muellbauer (1977). In those studies it is general equivalence scales, which are estimated. All the other scales referred to here concerns specific scales. The specific scales refer to dietary adequacy, food expenditure, expenditures on necessities or to savings.

Table 7.6. Comparison of different equivalence scales for the U.K., U.S.A. and Holland. The scales are normalised to unity for an adult childless couple. For the respective methods of estimation, see in the list of references.

Note. The Amsterdam scale is from Bureau van statistiek, Amsterdam 1917.

No. of children	Investigation	Muellbauer(1977) £50 per week U.K.	Friedman(1965) U.S.A.	Eason and Wentworth(1947) U.S.A.	The Amsterdam scale Holland
1		1.141	1.375	1.300	1.420
2		1.273	1.695	1.495	1.710
3		1.398	1.965	1.700	1.945
4		-	2.205	1.910	2.130

No. of children	Investigation	Seneca and Taussig(1971) $12000 U.S.A. Food	Seneca and Taussig(1971) $12000 U.S.A. Necessities	Nicholson(1965) U.K.	U.S. Bureau of Labor Statistics 1960-61 U.S.A.
1		1.460	1.408	1.310	1.365
2		1.761	1.558	1.550	1.665
3		1.963	1.696	1.750	1.935
4		2.188	1.847	1.935	2.200

No. of children	Investigation	U.S. Bureau of Labor Statistics 1946,1947 U.S.A.	Nicholson(1976) U.K.	Kapteyn and van Praag(1976) Holland
1		1.275	1.130	1.081
2		1.515	1.250	1.162
3		1.725	1.360	1.230
4		1.925	1.470	1.257

Since most of the previous studies have been concerned with specific equivalence scales, especially for food or necessities, it is quite natural that estimates of general equivalence scales appear lower. This is also the case here, for both our estimated scales. The equivalence scales based on demographic translation are intermediate here and rather close to the scales found by Eason and Wentworth (1947) which are estimated for a maintanence budget.

To summarise and conclude, we shall prefer the estimated equivalence scales based on the Barten cost function. We shall use these scales in computing inequality indexes for the expenditure distributions in Sweden 1969 and 1978. For theoretical reasons it is preferrable to use general instead of specific scales. The estimates here are close to those found in three other studies for other data sets. Secondly, the estimates have been obtained without too much Bayesian econometrics. This was not the case for the demographically translated demand system, where two important parameters were fixed according to official equivalence scales and where the final estimates appeared to be too much biased towards these official scales.

Therefore, in the next section we present cost of living indexes for Swedish data, explicitly based on equivalence scales emanating from the Barten cost function.

8 PRICES THE COST OF LIVING AND INEQUALITY INDEXES

8.1 Introduction

In the previous chapter we derived equivalence scales from the estimation of complete demand systems for Norwegian household budget data. Also, we estimated a demand system for Swedish time series data. In this chapter, we shall combine these two estimations in order to compute cost of living indexes for Swedish household with different utility levels and compute inequality indexes for Swedish utility distributions in 1969 and 1978, those years for which family expenditure surveys are available.

8.2 Cost of Living Indexes

In the family expenditure surveys we find data on x^h, total nominal expenditure of household h. To compute the cost of living index for different utility levels, we need data for x_N^h. To get x_N^h for the two different expenditure distributions in 1969 and 1978, we used the estimated parameters (based on Barten's cost function) for the Norwegian data and simulated equivalence scales for the Swedish total expenditures, x^h.

Recall again the definition of x_N^h in (4.13), i.e.

$$x_N^h = \frac{c(f(x^h,p,a^h),p,a^h)}{c(f(x^h,p,a^h),p^0,a^h)} \; x_*^h = \frac{c(f(x^h,p,a^h),p,a^0)}{c(f(x^h,p,a^h),p,a^h)} \; x^h .$$

The first RHS is x_*^h times a cost of living index and the second RHS is x^h times an equivalence scale. To get x_N^h, we simulated the latter equivalence scale for Swedish price and income data in 1969 and 1978. In addition, the parameter α_0 was changed to conform better with the Swedish data. x_N^h was then used to compute the cost of living index in the first RHS above and hence we arrived at the welfare measure x_*^h.

Table 8.1. Equivalence scales for the Swedish family expenditure surveys for 1969 and 1978, based on simulations with α_0 fixed to 9.903 for 1975, the year for which prices were normalised to unity.

Year	x^h	a_h	Barten equivalence scale	x_N^h
1969	17187	0	1.000	17187
	25856	0	1.000	25856
	30535	0	1.000	30535
	22209	1	1.135	19567
	28014	1	1.127	24857
	33850	1	1.121	30196
	46770	1	1.110	42135
	25124	2	1.267	19830
	28399	2	1.258	22575
	36592	2	1.240	29510
	44523	2	1.226	36316
	31311	3.26	1.424	21988
	30367	3.26	1.428	21265
	38174	3.38	1.415	26978
	52219	3.27	1.361	38368
1978	38406	0	1.000	38406
	43525	0	1.000	43525
	63212	0	1.000	63212
	80771	0	1.000	80771
	68579	1	1.122	61122
	77141	1	1.118	68999
	92919	1	1.112	83560
	65847	2	1.252	52593
	72874	2	1.244	58580
	85023	2	1.233	68956
	103909	2	1.219	85241
	74771	3.2	1.400	53408
	79134	3.2	1.393	56808
	90521	3.1	1.363	66413
	112230	3.2	1.350	83133

The equivalence scales and x^h_N for the family expenditure surveys in 1969 and 1978 are presented in Table 8.1.

The households in these two surveys are divided into 15 groups. It should be noted that the FES in 1969 excludes pensioners. The FES in 1969 covers about 1.3 million households whereas the FES in 1978 covers about 1.8 million households.

As immediately can be seen in Table 8.1 the distribution of x^h_N must be more equal than that of x^h, since x^h increases with the number of children. Note, however, that the equivalence scales, as before, decreases with x^h

The fact that the equivalence scales decrease with total expenditure implies that relative prices of goods of which children have relatively large needs have increased. As seen in Table 7.5 the specific equvialence scales are largest for Food, Clothing, Housing and Education, but low for Furniture, Transport and Miscellaneous. From Table 6.10 we see that the budget shares for Food, Clothing and Housing are much larger for poor households, while the budget shares for Furniture and Transport are much larger for "bliss" households.

Before looking at the estimates of cost of living indexes, it is instructive to look at the actual development of relative prices for the goods in our demand systems. The charts in Appendix 1 depict the relative-price changes for the period 1963-1979. According to the Swedish data Food, Clothing, Housing and Miscellaneous are necessities and Furniture, Transport and Education are luxuries. The relative prices have decreased for Clothing but have increased for Miscellaneous. The relative prices of Food and Housing seem to move almost inversely proportional to each other. Relative prices of Furniture have increased but relative prices of Education have decreased. Hence, there is no large differences in the changes of relative prices between necessities and luxuries.

Thus, it is likely that the cost of living indexes across utility levels will not differ very much. In Table 8.2 the cost of living indexes for different total expenditure, x^h_N, levels are presented.

Table 8.2. Cost-of-living indexes and real income measures in Sweden 1969 and 1978. The cost-of-living indexes are for the expenditure distributions in the Swedish family expenditure surveys in 1969 and 1978. The indexes are based on the expenditures x_N^h. Prices are normalised to unity in 1975.

Year	x_N^h	a_h	Cost of living index index	x_*^h
1969	17187	0	.64138	26797
	25856	0	.64067	40358
	30535	0	.64038	47683
	19567	1	.64116	30518
	24857	1	.64074	38794
	30196	1	.64040	47152
	42135	1	.63983	65853
	19830	2	.64113	30930
	22575	2	.64091	35223
	29510	2	.64044	46078
	36316	2	.64009	56736
	21988	3.26	.64095	34305
	21265	3.26	.64101	33174
	26978	3.38	.64061	42113
	38368	3.27	.63998	59952
1978	38406	0	1.36555	28125
	43525	0	1.36398	31910
	63212	0	1.35932	46503
	80771	0	1.35627	59554
	61122	1	1.35973	44952
	68999	1	1.35823	50801
	83560	1	1.35586	61629
	52593	2	1.36162	38625
	58580	2	1.36026	43065
	68956	2	1.35825	50768
	85241	2	1.35560	62881
	53408	3.2	1.36142	39230
	56808	3.2	1.36066	41750
	66413	3.1	1.35870	48880
	83133	3.2	1.35591	61312

The cost of living indexes are computed for the total expenditure levels in the two Swedish family expenditure surveys in 1969 and 1978. It is clear that the differences between the indexes are very small, but that the cost of living undoubtedly rises with the total expenditure level. In 1969 the cost of living index is .64138 for the lowest total expenditure level and .63983 for the highest, and the relative difference is only 0.24 per cent. In 1978 the cost of living index is 1.36555 for the lowest total expenditure level and 1.3556 for the highest. The relative difference is 0.73 per cent.

The differences across households thus are larger for the 1978 family expenditure survey. In Appendix 2 the cost of living indexes for the whole sample period 1963, for the different total expenditure levels in the both family expenditure surveys, are given. The largest difference between the cost of living of the lowest and the highest total expenditure level, for the 1969 FES, can be found in 1977 and is 0.95 per cent. For the 1978 FES this difference is 0.84 per cent. For the whole period 1963-1979 the cost of living has increased for the less well off: the difference between the lowest and the highest total expenditure brackets, is 0.89 per cent for the 1969 FES and 1.01 per cent for the 1978 FES. Hence, the differences in the cost of living across total expenditure levels are very small and, as can be seen in Appendix 2, for some years changes in relative prices have been egalitarian, though, for the period as a whole, it has been inegalitarian.

In Table 8.3 we have computed inequality indexes for the distributions of x^h, x^h_N and x^h_*. We use the inequality measures from Chapter 4, i.e., the coefficient of variation, Theil's entropy measure, the Gini coefficient and Atkinson's inequality index.

Table 8.3. Inequality indexes. The indexes are measured for the distributions of x^h, x^h_N and x^h_* in Sweden 1969 and 1978. C is the coefficient of variation, T is Theil's entropy measure, G is the Gini coefficient and A(ε) is Atkinson's inequality index.

Index	x^h		x^h_N		x^h_*	
	1969	1978	1969	1978	1969	1978
C	.24951	.27149	.21720	.22133	.21782	.22325
T	.03012	.03795	.02272	.02517	.02290	.02561
G	.13157	.15289	.11766	.12419	.11798	.12528
A(2)	.05890	.08061	.04340	.05315	.04363	.05410
A(3)	.08774	.12140	.06348	.08055	.06381	.08198
A(6)	.16933	.22145	.11669	.15462	.11726	.15710

As can be seen, inequality is much lower, when account is taken of the cost of children, and this reduction is substantial. The change in Atkinson's inequality index has a nice interpretation, since the social welfare function in this case is defined by

$$(8.1)\ SW = \bar{x}_* E = \bar{x}_* (1-A) ,$$

where is the equality index defined by (4.9). Hence, a decline in the inequality index A is equivalent with an equal relative change in in real income, $\bar{x}_*$, for everyone.

The effects of relative-price changes on inequality in 1969 and 1978 can also be seen in Table 8.3, as the difference in inequality between x^h_N and x^h_*

Table 8.3. Inequality indexes. The indexes are measured for the distributions of x^h, x^h_N and x^h_* in Sweden 1969 and 1978. C is the coefficient of variation, T is Theil's entropy measure, G is the Gini coefficient and A(ε) is Atkinson's inequality index.

Index	x^h		x^h_N		x^h_*	
	1969	1978	1969	1978	1969	1978
C	.24951	.27149	.21720	.22133	.21782	.22325
T	.03012	.03795	.02272	.02517	.02290	.02561
G	.13157	.15289	.11766	.12419	.11798	.12528
A(2)	.05890	.08061	.04340	.05315	.04363	.05410
A(3)	.08774	.12140	.06348	.08055	.06381	.08198
A(6)	.16933	.22145	.11669	.15462	.11726	.15710

Table 8.4. Price inequality indexes. The same indexes as in Table 8.4 are reported. The price inequality indexes are defined by (4.7), (4.8), (4.15) and (4.16).

Index	PI	
	1969	1978
C	1.0028545	1.0086748
T	1.0079225	1.0174811
G	1.0027197	1.0087769
A(2)	1.0052995	1.0178739
A(3)	1.0051985	1.0177529
A(6)	1.0048847	1.0160393

In Table 8.4 the indexes of price inequality, which are simply the ratios between the inequality indexes for x^h_* and x^h_N. As can be seen, inequality rises for all the different inequality indexes, and hence relative prices have been inegalitarian both in 1969 and 1978, the last year of which this effect has been strongest. To get an idea of the welfare implications of these effects we can consider the Atkinson social welfare function in (8.1).

The price inequality index PI then indicates the compensation needed in $\bar{x}_*$, i.e. the amount which should be transferred to everybody, in order to compensate for the change in inequality induced by changes in relative prices or, equivalently, to arrive at the level of social welfare which is associated with relative prices that do not affect inequality. Suppose we consider Atkinson's index with the parameter ε=2. This parameterisation implies that if a household X has twice the income of a household Y, then the additional welfare of another Skr (Swedish crown) for Y is four times that of another Skr for X.

As can be seen in Table 8.4, the price inequality index PI, defined for the inequality index A(ε), is not particularly sensitive to different values of the parameter ε, at least not for values in the range 2-6. Hence, with ε=2, to compensate for price inequality everybody's income should increase by 0.53 % in 1969 and by 1.79 % in 1978. Thus, considered in this way, the relatively small differences in price indexes across households, still imply considerable reductions in social welfare. In 1969 the reduction in money terms, in 1975 year's prices, is 270 million Skr, or 207 Skr for each of the 1.3 million households included in the 1969 FES. For 1978 the corresponding figures are higher, 1 500 million Skr, or 835 Skr for each of the 1.8 million households covered in the 1978 FES.

These exercises with a normative inequality measure maybe should not be taken too serious, but I still think that it should remind the reader that apparently small differences in cost of living indexes across households can have substantial welfare implications.

In conclusion, for the two years of expenditure data in Sweden, 1969 and 1978, relative prices have been inegalitarian. Cost of living indexes for the period 1963-1979 for different total expenditure levels, x^h_N, indicates that in the 1960s relative prices not seldomly the less well off have risen more. The use of a normative inequality measure, Atkinson's inequality index, shows that relative prices have been inegalitarian in 1969 and 1978, and that the social welfare loss of relative-price changes can be substantial.

9 INCOME DISTRIBUTION POLICY

Different measures are used in the official policy in order to adjust the distribution of income. Income and wealth taxes and different kinds of transfers are well-known examples of this kind of policy. As this study has shown, it would also be possible for the government to affect the distribution of income through a policy which aims at changing relative prices. If such a policy aims at reducing inequality, it should reduce relative-prices of necessities.

The purpose of this chapter is to examine the effects of two different kinds of income distribution policies, which have been pursued by the Swedish government in the 1970s, and compare the relative merits of the policies, taking government policy objectives as given. The first kind of policy is price subsidies of necessities, in particular of Food and Housing, which have been conducted by the Swedish government in the 1970s in order to, at least partly, reduce inequality. The second kind of policy is child allowances, which have been a fixed money transfer per child, independent of income. Though the actual motive behind these policies may have been others, the concern of the income distribution has been a central objective of the policy.

The aim here is to compare the effects on inequality of, on the one hand, price subsidies of Food and Housing, and, on the other hand, child allowances.

9.1 Price Subsidies and Inequality

Price subsidies became an important part of Swedish income distribution policy, especially in the 1970s. Then, price subsidies of Food and, in particular, of Housing, increased. Housing cost were subsidised by approximately 10 % (in 1983) and certain food prices by even more than that. For the Food component in CPI as a whole, prices were subsidised by approximately 5 %.

The objectives of these price subsidies have been to reduce inequality, particularly the Food subsidies, and to support families with children. There is also paternalistic aims behind the subsidies, such as "everybody should eat well" or "nobody should live in too small apartments".

In this section we shall consider price subsidies of Food and Housing, where these prices have been subsidised by 5 %, respectively. Since the aim is to compare the relative merits of different income distribution policies, the financing of them are left aside.

Consider again the social welfare function implied by Atkinson's inequality index:

$$SW = \bar{x}_* (1-A) ,$$

where A is the inequality index and $\bar{x}_*$ the mean real income. We are here mainly concerned with A, but x_* will change differently, depending on whether it is Food or Housing which are subsidised.

In order to compute x_N^h and x_*^h for the new price situations we have simulated new consumption patterns, based on the previous estimates of the complete demand systems, and computed new equivalence scales and cost of living index numbers, and hence, obtained x_*^h, for which new inequality indexes have been obtained.

In Table 9.1 the new inequality indexes for 1969 and 1978 are presented.

Table 9.1. Inequality indexes for x_*^h after price subsidies of Food and Housing with 5 per cent in Sweden 1969 and 1978.

Index	Food		Housing	
	1969	1978	1969	1978
C	.21633	.22161	.21760	.22297
T	.02259	.02529	.02285	.02555
G	.11712	.12423	.11788	.12512
A(2)	.04307	.05362	.04354	.05395
A(3)	.06302	.08141	.06368	.08175
A(6)	.11599	.15662	.11700	.15669

Comparing these to the inequality indexes before the price subsidies, as given in Table 8.3, we can see that inequality decreases, and that price subsidies of Food are more egalitarian than price subsidies of Housing. This result should not surprise us, since both Food and Housing are necessities according to the previous estimations.

As can also be seen by comparing tables 8.3 and 9.1, subsidies of Food more than compensates for the observed price inequality in 1969, but not in 1978, which is shown by comparing the inequality indexes after the price subsidies with the inequality indexes for the actual distribution of x_N^h in Table 8.3. A 5 % price subsidy of Housing does not compensate for the observed price inequality, neither in 1969 nor in 1978.

Again, these results indicate that the observed effects on inequality of relative-prices in 1969 and 1978 are substantial.

9.2 Child Allowances and Inequality

Whereas price subsidies of necessities inevitably will affect the distribution of income in an inegalitarian direction, child allowances in general only will do that if families with children have relatively low incomes. If, by some reason, that is not the case, the effect will go in the opposite direction.

In the public debate in Sweden, it has been strongly asserted that the standard of living of families with children are too low compared to childless families, and that subsidies to children should increase. The most recent measures taken by the government[1] involve a large increase in child allowances, while at the same time price subsidies of Food are decreased. In this section we shall examine what the effects on the distribution of income are, from such policy measures.

The effects on real total expenditure, x_*^h, of a 5 % price subsidy of Food and Housing, respectively, are that $\bar{x}_*$, the mean real total expenditure, increases: if Food is subsidised, with 851 Skr in 1969 and 1029 Skr in 1978 and if Housing is subsidised, with 329 Skr in 1969 and 381 Skr in 1978, all in 1975 year's prices. Hence, a 5 % subsidy of the food prices is worth considerably more than a 5 % subsidy of housing cost, since food takes a larger share of total expenditure.

To compare price subsidies to child allowances we compute the child allowances so that $\bar{x}_*$ is equal in the respective cases, i.e. the $\bar{x}_*$ corresponding to price subsidies of Food implies larger child allowances than the $\bar{x}_*$ associated with Housing subsidies. This is a straight-forward application of Atkinson's social welfare function: keeping $\bar{x}_*$, differences in social welfare, implied by the different income distribution policies, only depend on their effects on inequality.

Table 9.2 gives the new inequality indexes, when child allowances have replaced price subsidies of Food and Housing, respectively.

Table 9.2. Inequality indexes for x^h_* when price subsidies for Food and Housing have been substituted with direct subsidies for children.

Index	Instead of price subsidy of:			
	Food		Housing	
	1969	1978	1969	1978
C	.21584	.22894	.21680	.22507
T	.02255	.02710	.02271	.02610
G	.11521	.12802	.11688	.12627
A(2)	.04340	.05796	.04343	.05541
A(3)	.06395	.08823	.06369	.08413
A(6)	.12049	.16948	.11819	.16154

Comparing tables 8.3, 9.1 and 9.2, we actually find that that child allowances <u>increase</u> inequality. If child allowances are used instead of food subsidies, inequality increases by 0.77 % in 1969 and with 8.09 % in 1978, if we compare Atkinson's inequality index with $\varepsilon=2$. In terms of social welfare, the reduction in 1978 is remarkable and corresponds to a loss in social welfare of 7 000 million Skr.

If child allowances are used instead of housing subsidies, the results are similar, though in this case, for some of the inequality indexes in 1969, inequality is <u>reduced</u>. However, in 1978 child allowances instead of housing subsidies increase inequality by 2.71 %, again, in terms of Atkinson's social welfare function, representing a considerable loss in social welfare.

Finally, if we compare actual inequality in Table 8.3 with inequality implied by the imposed child allowances in Table 9.2, we find that child allowances <u>decreased</u> inequality slightly in 1969, but <u>increased</u> inequality substantially in 1978.

In conclusion, the results here show that recent Swedish policy, implying huge increases in child allowances and decreases in food subsidies, will <u>increase</u> inequality. The implication is also that this policy is misguided, since the aim of the policy was to <u>reduce</u> inequality.

9.3 Taxes and Family Policy

All the policy measures discussed in the two previous sections are part of which constitutes Swedish family policy. As such, the independent effects of such policies are interesting to study, but it may well be that the different policies are mutually dependent. For instance, some policies may be politically more attractive than others, and hence the composition of income distribution policy may be of a complex nature.

The main tool for redistributing income is the progressive income tax system. However, since the cost of children is not deductable from income, the Swedish income tax system acts so as to redistribute the incomes corresponding to x^h. However, as we argued in Chapter 3, x_N^h is a more appropriate welfare measure, since it accounts for household size, specifically the costs for maintenance of children.

If income distribution policy is designed to redistribute x_N^h, then tax rates should be a function of x_N^h instead of x^h, i.e. :

$$(9.2)\quad t = \Gamma(x_N^h) = \Gamma(x^h - (c(u,p,a^h) - c(u,p,a^0))) \ ,$$

where t is the tax rate, x^h is the total nominal expenditure of a cost-minimising reference household and $c(u,p,a^h) - c(u,p,a^0)$ is the cost of children, u is the utility level of a reference household and p is reference prices. However, the tax system in Sweden was changed in the early 1970s from joint filing to separate filing, in order to increase female labour force participation rates, so that tax rates depend on individual income and not on household income.

According to our previous analysis the first-best solution would be to use the tax system given by (9.2). That would, however, give rise to an increase in inequality, an increase which would be even bigger than the one implied by the child allowances in Table 9.2[2]. From the point of view of the Swedish government, inequality is too large. In order to reduce inequality, however, a <u>decrease</u> in child allowances may be the second-best solution.

The reason for this conclusion is that real total expenditure, x_*^h, in general is relatively higher for families with children.

9.4 Some Qualifications

In this study we have used total real expenditure adjusted for the cost of children, x_*^h, as a measure of welfare, or as a measure of the standard of living from the market basket of goods. Objections can be raised against this, e.g. since we have neglected household saving.

If certain households save more than others, it is clear that their incomes have been underestimated. On the other hand, the argument of the life cycle hypothesis implies that consumption is held constant over the life cycle and thus that the level of total consumption at a point in time fairly well reflects the average income over the life time. However, since the credit markets are not perfect, this argument may be a false one.

Another objection to the welfare measure x_*^h is that it does not include leisure time. Hence, if the parents are forced to work more than childless couples in order to maintain the same standard of living, we overestimate the welfare of families with children. There is also a reason for for parents to work less than childless couples, since it takes time to bring up and educate children. There is, however, no evidence in our data that participation rates decreases with the number of children, but rather seem to be independent of household size.

A reservation should also be made for the estimates of the equivalence scales. In Table 6.1 we also see that the estimated parameters δ_i in the equivalence scales show up with large standard errors. However, the comparison of different scales in Table 7.6 shows that the estimated equivalence scales, based on Barten's cost function, are close to previous estimates of general equivalence scales for other countries. Other estimated scales, such as the Amsterdam scale, are specific scales referring mostly to food or necessities. The estimated specific scales for Food and Clothing in this study is 1.175 and 1.605, respectively, and are larger than the estimated general scale. Therefore, the estimated equivalence scales seem fairly plausible.

Given these estimated scales, Table 8.2 shows that x_*^h increases with the number of children. Thus, the conventional wisdom, that families with children in general have a lower standard of living seems to be wrong.

We have here described the welfare level as a function of the number of children, but the causality could also go in the opposite direction, i.e. the number of children in the household depend on income. It is not unreasonable that households with relatively high incomes have more children, just as they have bigger houses and faster cars.

Finally, a reservation for the quality of data should be made. The family expenditure survey data were primarily collected from the households through book-keeping. Approximately 25 % of the households sampled for the book-keeping. Although this participation rate is low, the FES consumption data is regarded as fairly reliable and is used to revise the consumption data in the national accounts.

10 SUMMARY AND CONCLUSIONS

This study has analysed the effects of changes in relative prices on the distribution of income in Sweden. Since consumption patterns differ with income insofar that necessities takes a larger share of total consumption of the relatively poor households, changes in relative prices of necessities and luxuries affect the distribution of income.

In Chapter 2 two different approaches in the empirically oriented literature are discussed. The first is based on computing mechanical price indexes, like the Laspeyeres and Paasche indexes, for different consumption patterns, and the second approach, which is the one used in this study, is based on the economic theory of the consumer.

Chapter 3 describes the theory of the consumer and defines two important concepts: the cost of living index and the equivalence scale. The powerful method of duality in the theory of the consumer is used, where the demand functions are derived from the cost minimisation problem of the consumer.

Household preferences are introduced as the preferences of adult couples and cost of living indexes and equivalence scales are defined within this context. Household utility and the minimum cost of a certain utility level is shown to depend on the number of children in the household. Household utility is increasing in the vector of consumption goods, but decreasing in the vector of household size. The foundation of the utility and cost functions are discussed and it is shown that it is a realistic assumption that utility is separable in the market basket of goods and the number of children and that the equivalence scale consistently reflects the cost of children.

Given the concept of household preferences it is straight-forward to derive cost of living indexes and equivalence scales. It is shown that the cost of living index is a ratio between two cost functions, showing the relative change in minimum cost, necessary to maintain a certain utility level given household size (the number of children), due to changes in prices.

The equivalence scale is shown to be a concept close to the cost of living index, showing the relative change in minimum cost, necessary for a large household to be as well off as a small household, given prices.

In Chapter 4 a welfare measure is derived, based on the concept of household preferences. This welfare measure is a measure of utility from the market basket of goods, which takes account of the cost of children and to differences in the cost of living across households. Different inequality indexes applied to this welfare measure are discussed, and their relationship to social welfare functions. A price inequality index is derived, which shows the necessary relative change in average welfare, needed to compensate for the change in inequality induced by changes in relative prices.

Chapter 5 gives empirical specifications and the data to be used. The computation of cost of living indexes and equivalence scales is achieved through the estimation of complete systems of demand functions. Use is made of the duality in consumer theory and the preferences are specified in the cost function. The specification used is the AIDS (Almost Ideal Demand System), which is due to Deaton and Muellbauer, a demand system which uses a flexible functional form approach which is a second-order approximation to an arbitrary cost function.

The estimation of equivalence scales presupposes household budget data. Since this study is concerned with price inequality in Sweden and there are only two family expenditure surveys available in Sweden, we have used Norwegian data in the computation of equivalence scales. The Norwegian parameters are then used to simulate equivalence scales for Swedish expenditure data. The data are annual consumption expenditures of seven aggregate commodities: Food, Clothing, Housing, Furniture, Transport, Education and Miscellaneous. The family expenditure surveys in Norway are from 1967 and from 1973-1979.

The estimation of the equivalence scales were done in two alternative ways, through demographic scaling and translation, both of which transform the cost function to account for the number of children in the household. The demographic scaling, based on Barten's cost function, turns out to be easier to interpret and gives reasonable translation implied implausible estimates of the equivalence scales when estimation was done without priors for some of the parameters.

Chapter 6 gives the empirical results of the estimated demand systems, for family expenditure data in Norway, and for time series data for Sweden for the period 1963-1979. The estimates turns out to be reasonable and the goodness of fit is high, compared to earlier empirical results for Swedish data.

The estimated equivalence scales are presented in Chapter 7. The average cost of one child decreases in utility levels, and is in the range 5 - 10 % of an adult couple. The average cost of three children is in the range 12 - 26 % and hence there are some economies of scale in children.

In Chapter 8 inequality indexes are computed for the total expenditure distributions in Sweden 1969 and 1978. The price inequality indexes show that relative prices in 1969 and 1978 have been inegalitarian, i.e. prices of necessities were relatively high. The inegalitarian bias in relative prices is higher in 1978 than in 1969 and is approximately 1.5 % and 0.5 %, respectively. Interpreted in terms of the social welfare function, this inegalitarian bias in prices implies a relative decline of average social welfare of the same size, and hence need not be negligable.

The estimates of cost of living indexes for the total expenditure distributions in 1969 and 1978, for the period 1963-1979, also show that the differences in the cost of living across utility levels have increased in the 1970s.

The inegalitarian bias in relative prices can be counter-acted by a policy which subsidises necessities. This kind of policy have been pursued by the Swedish government through price subsidies of Food and Housing. In Chapter 9, we simulate new consumption patterns, equivalence scales and cost of living indexes, based on a 5 % subsidy of Food and Housing, respectively. The inequality indexes show that these subsidies both are egalitarian. As an alternative to these kind of subsidies, the Swedish government recently has proposed a policy in which child allowances are raised and price subsidies of Food are decreased. According to the government this policy measure was used in order to equalise the distribution of income. However, as our inequality indexes show, increasing child allowances actually will <u>increase</u> inequality, contrary to the conventional wisdom. The reason for that is that utility levels - with account taken for the cost of children - in general are higher for families with children.

In conclusion, this study has shown that relative-price changes in Sweden in 1963-1979 in general have been inegalitarian. Inequality increased by 0.5 % in 1969 and by 1.5 % in 1978, due to changes in relative prices. It is also shown that price subsidies of necessities may be a useful tool in order to affect inequality and that child allowances actually may increase inequality.

NOTES

Chapter 1

1) See Sheshinsky and Weiss (1977), Fischer (1981), Cukierman (1983), and Assarsson (1984).

Chapter 2

1) See Fleetwood(1745), ch. IV.

2) On this problem, see Voeller and Eichhorn(1976) and Pollak(1971). Pollak shows that the true cost of living index is a function of price relatives only if the preferences is represented by the Cobb-Douglas utility function.

Chapter 3

1) A full description of the solution is given in a standard textbook, as e.g. Malinvaud(1971).

2) A good survey of duality in consumer theory is given in Deaton and Muellbauer(1980b).

3) A proof of the existence of c(u,p), the so called integrability condition, can be found in Hurvicz and Uzawa(1971).

4) For proofs, see Fuss and McFadden(1978).

5) For a proof, see Shephard(1953,1979), Diewert(1974) or Deaton and Muellbauer(1980b).

6) $\delta_{ij} = 0$ for $i = j$

$= 1$ for $i \neq j$.

Chapter 4

1) On the problems of how to arrange individual preferences into social choice, see Arrow(1951) or Sen(1970). Sen(1973) gives a comprehensive discussion of different measures of inequality.

The following analysis rests heavily on Muellbauer(1974a)

3) The variance and the coefficient of variation are common statistical measures. The Gini coefficient was proposed by Gini(1912) and the entropy measure in economics by Theil(1967).

Chapter 5

1) See Klein and Rubin(1948) or Geary(1950).

2) On the translog model, see Christensen, Jorgensen and Lau(1975).

3) On the Rotterdam model, see Theil(1976,1980). On empirical applications of this model, see Barten(1969) or Theil(1971).

4) On the aggregation properties of the PIGLOG cost function, underlying the AIDS, see Muellbauer(1975,1976). The translog model based on a specification of the indirect utility function (see Christensen, Jorgensen and Lau(1975)) is not so easy to estimate, and the specification of the direct utility function implies the assumption that prices are determined by quantities, rather than the reverse. Furhtermore, in general the translog model does not permit consistent aggregation; see Muellbauer(1975) and Deaton and Muellbauer(1980a).

5) See Assarsson(1983), which gives a brief comparison of the application of AIDS to Swedish data, to the extensive results of other demand systems, done in Klevmarken(1981).

6) Exact nonlinear aggregation is defined by Muellbauer(1975) in terms of the existence of a representative consumer. Such a consumer exists if

$$\bar{w}_i = w_i(u_0,p) = \frac{\partial \log c(u_0,p)}{\partial \log p_i} = \sum_h \frac{x_h}{\sum_k x_k} \frac{\log c_h(u_h,p)}{\log p_i}$$

where $c(u_0,p)$ is the cost function of the representative consumer.

7) If the original demand system is consistent with economic theory, so will the translated be. However, according to Pollak and Wales(1980, p. 596), the modified system satisfies the Slutsky symmetry condition, but the matrix need not be negative semi-definite, unless the d 's are zero.

8) Actually, the Swedish national accounts were changed in 1963, dividing the sample into two more or less homogenous subperiods. To obtain an almost homogenous set of data over the entire period, 1950-1979, individual items in the commodity groups were rearranged. Some commodities, such as health care, were deleted, since the definitions had been changed during the sample period.

Chapter 6

1) The well-known Central Limit Theorem states that the limiting distribution of a sum of a sequence of identically distributed random variables is normal, whatever the common distribution of these random variables.

2) The iteration method and the algorithm is described in Berndt et. al. (1974).

3) Note here that the parameter α_0 is fixed. This makes the estimation easier because of the nonlinearities which can be avoided in the estimation. Experiments with α_0 as a free parameter was not very promising, with α_0 taking implausible values. Since α_0 is interpreted as <u>subsistence total expenditure at reference prices</u>, it is reasonable to fix α_0 at some value. We have chosen the value 8.987 as the subsistence total expenditure for a childless adult couple in 1974, which corresponds to 8000 Norwegian crowns, which is somewhat below the lowest stratum for that demographic profile in the Norwegian family expenditure survey. The choice of α_0 is somewhat a matter of

4) See Berndt et. al. (1974), section 5. In multivariate regression the derivatives of the model with respect to the parameters do not depend on the endogenous variables. This implies that the Jacobian matrix of the transformation from the underlying disturbances to the observed endogenous variables, equals unity. Hence, the log of the likelihood function is simply a constant minus the log of the determinant of the variance-covariance matrix of the residuals.

5) Note that the standard result is that homogeneity is rejected. (See, i.a. Barten (1969), Byron (1970), Deaton (1974) or Deaton and Muellbauer (1980)). Observe, however, that the test for homogeneity has been shown to be biased towards rejecting homogeneity, which has been shown by Laitinen (1978).

6) See Barten (1969), Byron (1970), Deaton (1974) or Theil (1976):

7) Actually, the estimations were done for two different time periods, 1963-79 and 1950-79. Though the degrees of freedom are less for the shorter period, we preferred it to the longer period. The reason for that was that the data for total expenditures, which are used to evaluate the price effects on the distribution of income, are from 1969 and 1978. Also, the quality of data is much better for the shorter period, due to redefinitions of the items during the longer sample period.

8) The constant elasticity of demand system is one of the most widely applied demand systems. It is due to Schultz (1938) and Wold (1952). The linear expenditure system emanates from Stone (1954) and has been extended to include habit formation by Pollak and Wales (1969) and Pollak (1970). For an application of the latter on Swedish data, see Dahlman and Klevmarken (1971).

Chapter 7

1) For a summary of different kind of scales with reference to different measured scales, see Nicholson (1976).

2) This approach is associated with those which base the scales on particular commodities, notably <u>food</u> or <u>necessities</u>. See Friedman (1965) and Seneca and Taussig (1971).

3) The methods used in these surveys are described in van Praag (1971) and Kapteyn and van Praag (1973).

Chapter 9

1) A decision to raise the child allowances was taken in Spring 1984, which was supposed to be implemented in January 1985. Child allowances were increased by 1500 Skr per year and child, with an additional allowance for families with more than two children.

2) That is the implication for two reasons. First, income tax rates are progressive and hence the deduction is absolutely larger for households with high incomes. Secondly, as can be seen in Table 7.4, the compensating variation, given by $c(u,p,a^h) - x^h$, increases with income.

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APPENDICES

APPENDIX 1: RELATIVE PRICES 1963-1979

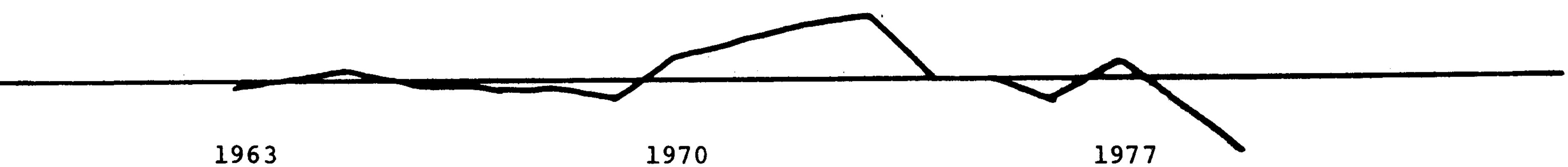

Figure 1. Relative price of Food 1963-1979.

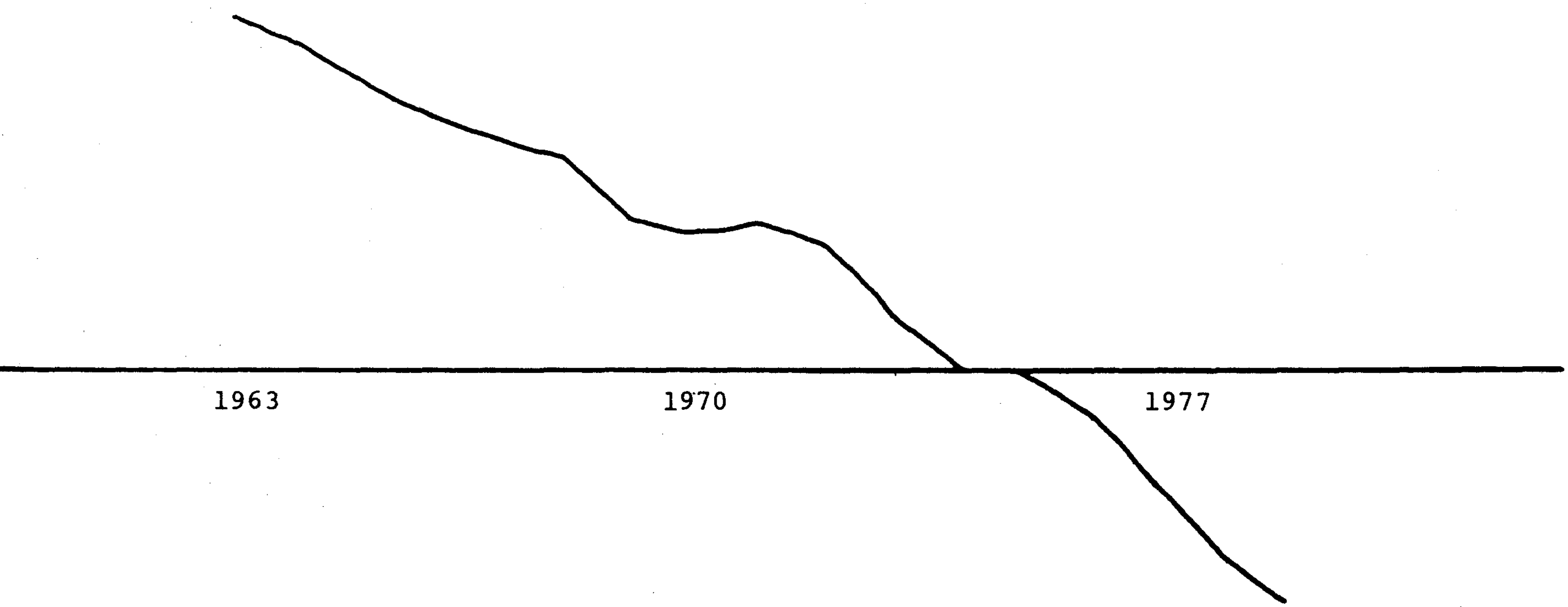

Figure 2. Relative price of Clothing 1963-1979.

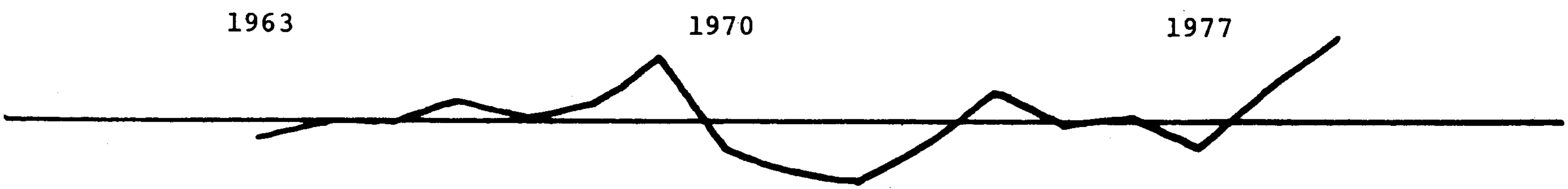

Figure 3. Relative price of Housing 1963-1979.

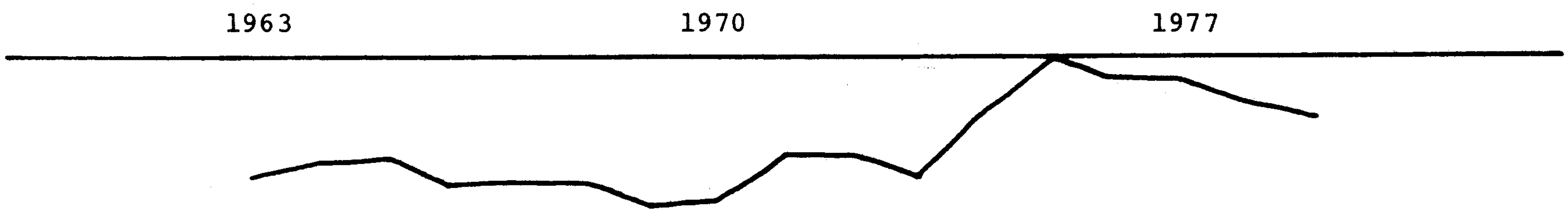

Figure 4. Relative price of Furniture 1963-1979.

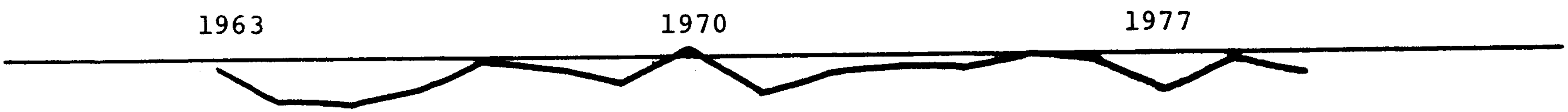

Figure 5. Relative price of Transport 1963-1979.

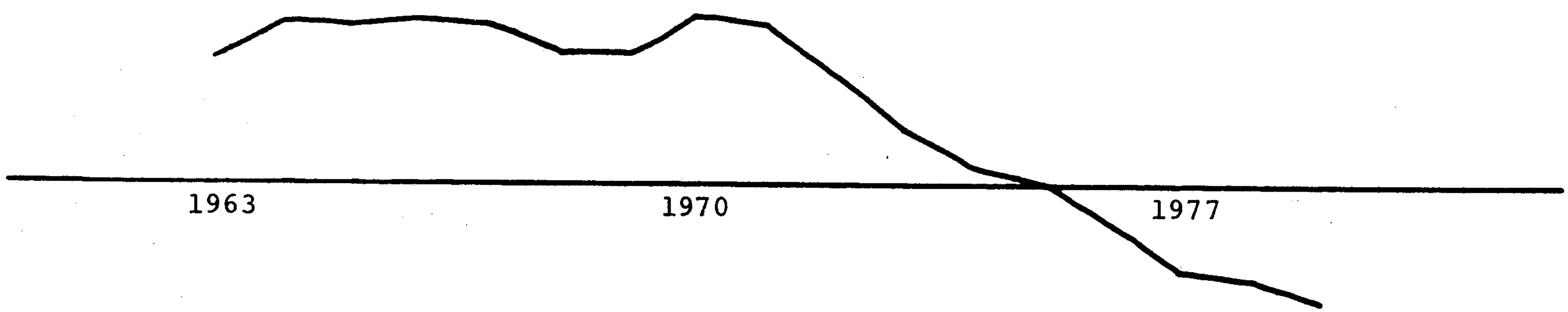

Figure 6. Relative price of Education 1963-1979.

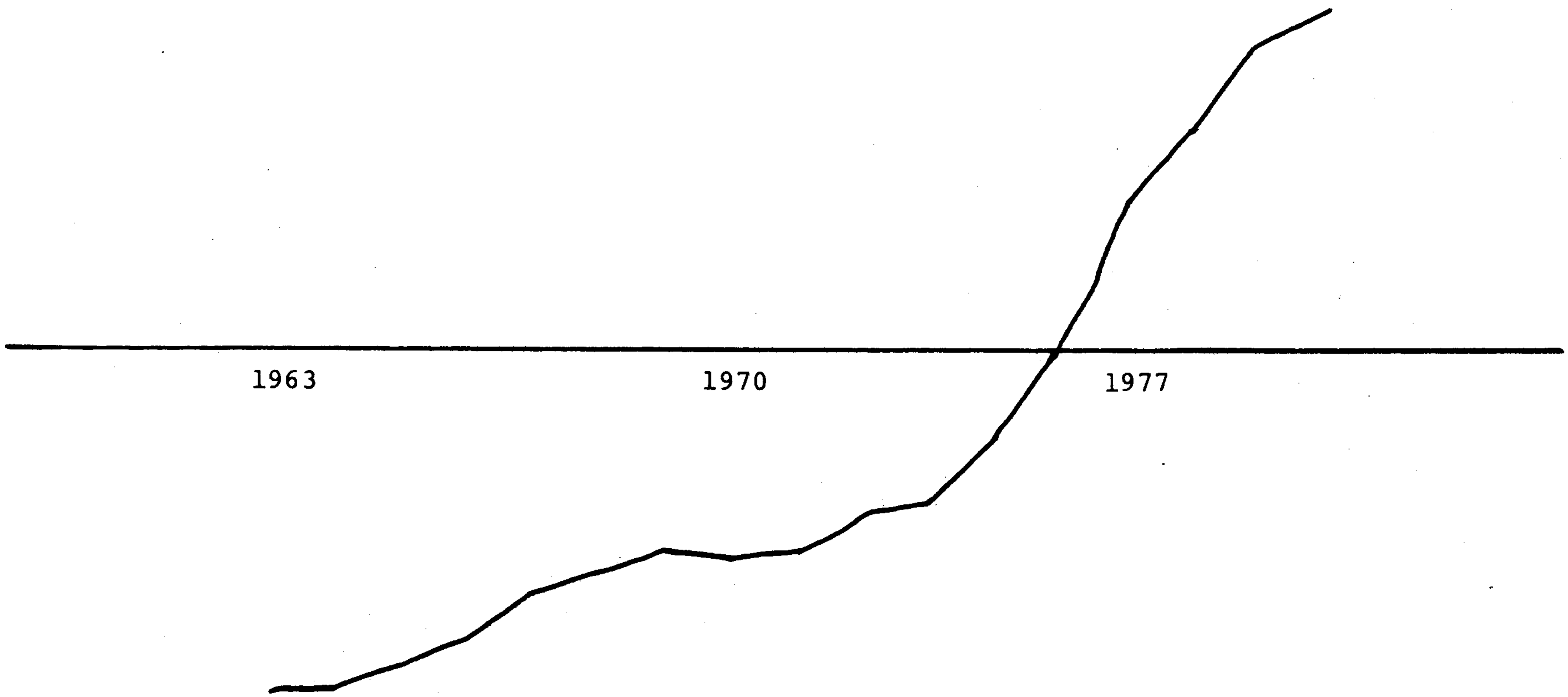

Figure 7. Relative price of Miscellaneous 1963-1979.

APPENDIX 2: COST OF LIVING INDEXES 1963-1979

For the total expenditure distributions in 1969 and 1978

Cost of Living indexes for total expenditures, x_N^h in 1969.

Time x_N^h	17187	19567	19830	21265	21988	22575
1963	.50116	.50133	.50135	.50144	.50148	.50152
1964	.51883	.51898	.51899	.51908	.51912	.51915
1965	.54752	.54762	.54763	.54769	.54772	.54774
1966	.58340	.58346	.58347	.58350	.58351	.58353
1967	.60810	.60838	.60841	.60856	.60863	.60868
1968	.61830	.61834	.61834	.61836	.61837	.61837
1969	.64138	.64116	.64113	.64101	.64095	.64091
1970	.66197	.66226	.66229	.66245	.66253	.66259
1971	.71688	.71707	.71709	.71720	.71725	.71729
1972	.76570	.76568	.76568	.76567	.76567	.76566
1973	.82329	.82277	.82272	.82244	.82231	.82220
1974	.90272	.90260	.90259	.90253	.90250	.90248
1975	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1976	1.11389	1.11324	1.11317	1.11283	1.11266	1.11252
1977	1.23549	1.23380	1.23364	1.23273	1.23230	1.23196
1978	1.36626	1.36463	1.36447	1.36360	1.36317	1.36285
1979	1.46176	1.46015	1.45999	1.45912	1.45870	1.45838

Time x_N^h	24857	25856	26978	29510	30196	30535
1963	.50164	.50170	.50176	.50187	.50190	.50192
1964	.51926	.51931	.51936	.51946	.51949	.51950
1965	.54781	.54784	.54788	.54794	.54796	.54797
1966	.58357	.58359	.58361	.58365	.58366	.58366
1967	.60889	.60898	.60907	.60926	.60931	.60933
1968	.61840	.61841	.61843	.61845	.61845	.61845
1969	.64074	.64067	.64061	.64044	.64040	.64038
1970	.66280	.66289	.66230	.66319	.66324	.66327
1971	.71743	.71749	.71756	.71769	.71772	.71774
1972	.76565	.76565	.76565	.76563	.76563	.76563
1973	.82182	.82166	.82150	.82113	.82104	.82100
1974	.90239	.90236	.90233	.90224	.90222	.90221
1975	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1976	1.11204	1.11184	1.11164	1.11118	1.11107	1.11101
1977	1.23071	1.23021	1.22967	1.22850	1.22820	1.22806
1978	1.36164	1.36115	1.36063	1.35950	1.35921	1.35908
1979	1.45718	1.45670	1.45618	1.45506	1.45477	1.45464

Time x_N^h	36316	38363	42135
1963	.50215	.50221	.50234
1964	.51970	.51976	.51988
1965	.54811	.54814	.54822
1966	.58374	.58376	.58381
1967	.60970	.60981	.61002
1968	.61837	.61851	.61854
1969	.64009	.63998	.63983
1970	.66366	.66378	.66400
1971	.71800	.71807	.71822
1972	.76561	.76560	.76559
1973	.82031	.82008	.81971
1974	.90206	.90200	.90193
1975	1.00000	1.00000	1.00000
1976	1.11015	1.10986	1.10940
1977	1.22583	1.22511	1.22392
1978	1.35692	1.35622	1.35507
1979	1.45250	1.45180	1.45066

Cost of living indexes for total expenditures, x_N^h in 1978.

Time x_N^h	38406	43525	52593	53408	56808	58580
1963	.50123	.50140	.50165	.50166	.50175	.50179
1964	.51889	.51904	.51926	.51928	.51936	.51938
1965	.54757	.54767	.54782	.54782	.54788	.54789
1966	.58343	.58349	.58357	.58357	.58361	.58361
1967	.60822	.60849	.60890	.60893	.60906	.60912
1968	.61832	.61835	.61840	.61840	.61843	.61843
1969	.64128	.64107	.64074	.64071	.64060	.64055
1970	.66210	.66238	.66281	.66284	.66299	.66305
1971	.71696	.71715	.71744	.71745	.71755	.71759
1972	.76569	.76568	.76566	.76565	.76565	.76563
1973	.82306	.82256	.82181	.82174	.82151	.82137
1974	.90267	.90256	.90239	.90237	.90233	.90229
1975	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1976	1.11361	1.11298	1.11203	1.11195	1.11165	1.11148
1977	1.23475	1.23313	1.23069	1.23048	1.22970	1.22929
1978	1.36555	1.36398	1.36162	1.36142	1.36066	1.36026
1979	1.46106	1.45951	1.45716	1.45696	1.45621	1.45582

Time x_N^h	61122	63212	66413	68956	68999	80771
1963	.50184	.50189	.50195	.50201	.50200	.50221
1964	.51943	.51948	.51953	.51958	.51958	.51976
1965	.54792	.54795	.54799	.54803	.54802	.54814
1966	.58363	.58365	.58367	.58370	.58369	.58376
1967	.60921	.60929	.60939	.60948	.60947	.60981
1968	.61843	.61845	.61846	.61848	.61847	.61851
1969	.64047	.64042	.64033	.64027	.64027	.63999
1970	.66314	.66322	.66333	.66343	.66342	.66378
1971	.71765	.71771	.71778	.71784	.71784	.71807
1972	.76563	.76563	.76562	.76563	.76562	.76560
1973	.82120	.82108	.82088	.82074	.82073	.82010
1974	.90225	.90223	.90218	.90216	.90215	.90201
1975	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1976	1.11127	1.11111	1.11086	1.11069	1.11067	1.10988
1977	1.22873	1.22832	1.22768	1.22721	1.22719	1.22516
1978	1.35973	1.35932	1.35870	1.35825	1.35823	1.35627
1979	1.45528	1.45488	1.45427	1.45382	1.45380	1.45185

Time x_N^h	83133	83560	85241
1963	.50225	.50226	.50228
1964	.51979	.51981	.51983
1965	.54816	.54817	.54819
1966	.58367	.58378	.58379
1967	.60987	.60989	.60993
1968	.61852	.61853	.61853
1969	.63994	.63994	.63990
1970	.66384	.66386	.66390
1971	.71811	.71813	.71815
1972	.76559	.76560	.76559
1973	.81998	.81997	.81988
1974	.90198	.90199	.90196
1975	1.00000	1.00000	1.00000
1976	1.10974	1.10972	1.10961
1977	1.22479	1.22473	1.22447
1978	1.35870	1.35586	1.35560
1979	1.45149	1.45144	1.45119