When both wages and prices are sticky

Previously, in the basic models, only product prices were allowed to be sticky. In practice, it is possible that other prices are sticky as well. In addition, some prices might be more or less sticky than others and this could be important for the dynamics in the economy as a whole. For instance, producer prices, consumer prices, product prices, prices on services, wages, prices on tradeables, asset prices, etc. may differ substantially with respect to stickiness. In this section we therefore extend the basic model to include sticky wages. It is well-known that wages, particularly for labor with non-standard jobs, are sticky and that labor contracts may have a long duration. In Sweden, for instance, it is presently common with 3-year contracts. The model presented here was originally developed by (Erceg, C. J., D. W. Henderson and A. T. Levin 2000) and a somewhat simplified version of their model is described in (Gali, J. 2008), who is followed here. The sticky wages follow the same model from (Calvo, G. A. 1983) that was applied for producer prices in the basic model.

This means that there is monopolistic competition in the labor market and that the households supply differentiated labor to the firms, analogous to the firms supplying differentiated products to the consumers. Each period a constant fraction of the households are allowed to change their nominal wage. Consequently, the aggregate nominal wage responds sluggishly to shocks, implying inefficient markups on wages. Also, analogously with the product markets, relative wages change in response to nominal shocks and create an inefficient allocation of labor. For the central bank, there is now the additional problem of how to best counteract these inefficiencies.

The model with sticky wages and prices

Firms

A continuum of firms is assumed, similar to the basic model:

$$Y_t(i) = A_t N_t(i) \tag{1}$$

where $N_t(i)$ is an index of labor input used by firm i and defined by

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{1-\frac{1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \tag{2}$$

where $N_t(i,j)$ denotes the quantity of the type j labor employed by firm i in period t. The parameter ε_w is the substitution elasticity among labor varieties. Let $W_t(j)$ be the nominal wage for type j labor in period t. Wages are set by workers, or by unions that represent them, of each type and these wages are taken as given by firms. Firms minimize cost and choose the demand of labor of each type, given the firms' total employment (output) and is obtained as the partial derivative of the cost function with respect to wage of the jth type of labor as

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t(i) \tag{3}$$

where

$$W_t = \left[\int_0^1 W_t(i,j)^{1-\varepsilon_w} dj \right]^{\frac{1}{1-\varepsilon}} \quad (4)$$

is an aggregate wage index. Substituting (3) into (2) yields the wage bill

$$\int_0^1 W_t(j) N_t(i,j) dj = W_t N_t(i)$$

as the product of the wage index and the employment index. The firms then solve the same problem as in the basic model, i.e.

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \psi_{t+k} \left(Y_{t+k|t} \right) \right) \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon_p} C_{t+k}$$

where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$ is the stochastic discount factor for nominal payoffs, $\psi_{t+k}(.)$ is the cost function and $Y_{t+k|t}$ denotes the output in period t+k for a firm that last reset its price in period t. As in the basic model a Phillips curve can be derived as

$$\pi_t^p = \beta E_t \left\{ \pi_{t+1}^p \right\} - \lambda_p \hat{\mu}_t^p \quad (5)$$

where $\hat{\mu}_t^p \equiv \mu_t^p - \mu^p = -(mc_t - mc)$ and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}$.

Households

As in the basic model the typical household maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U \left(C_t(j), N_t(j) \right) \right\}$$

subject to a sequence of budget constraints where $N_t(j)$ is the quantity of labor supplied and

$$C_t(j) \equiv \left(\int_0^1 C_t(i,j)^{1-\frac{1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \quad (6)$$

where i refer to the type of good and j refer to the type of labor that the household is specialized to supply.

In each period a fraction θ_p reset their wage, while a fraction $1-\theta_p$ keep their wage fixed.

Optimal wage setting

Consider the household that optimize and chooses the wage W_t^* in order to maximize

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k U\left(C_{t+k|t}, N_{t+k|t}\right)\right\} \tag{7}$$

the expected discounted utility from consumption and leisure (disutility from working) during the period during which the wage is expected to be fixed at the level W_t^* set in the current period. (7) is maximized subject to the constraints given by the demand functions (3) and the flow budget constraints that are effective while W_t^* is. The optimality condition can be written

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{N_{t+k|t}U_c\left(C_{t+k|t}, N_{t+k|t}\right)\frac{W_t^*}{P_{t+k}} + \mathrm{M}_w U_n\left(C_{t+k|t}, N_{t+k|t}\right)\right\}$$

where $\mathrm{M}_w = \dfrac{\varepsilon_w}{\varepsilon_w - 1}$ is the wage markup and be rewritten as

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{N_{t+k|t}U_c\left(C_{t+k|t}, N_{t+k|t}\right)\left(\frac{W_t^*}{P_{t+k}} - \mathrm{M}_w MRS_{t+k|t}\right)\right\} \tag{8}$$

where $MRS_{t+k|t} \equiv -\dfrac{U_n\left(C_{t+k|t}, N_{t+k|t}\right)}{U_c\left(C_{t+k|t}, N_{t+k|t}\right)}$ is the marginal rate of substitution between consumption and hours worked in period t+k for the household that resets its wage in period t. In the special case of full wage flexibility, $\theta_w = 0$, we have $\dfrac{W_t^*}{P_t} = \dfrac{W_t}{P_t} = \mathrm{M}_w MRS_{t|t}$. That means that the wage markup is the wedge between the real wage and the marginal rate of substitution in the absence of wage rigidities. Log-linearising around the steady state yields the following approximate wage setting rule

$$w_t^* = \mu^w + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{mrs_{t+k|t} + p_{t+k}\right\} \tag{9}$$

where $\mu^w = \log \mathrm{M}_w$. The wage equation is increasing in expected future prices, which reflects that households are concerned about their purchasing power. The wage is also increasing in the marginal rate of substitution of labor (disutility of work) in terms of goods over the life of the set wage, because households adjust their expected real wage, given expected future prices. It can also be seen as depending on the value of the marginal disutility of working hours in terms of goods. As in the basic model the specification

$$U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$$

which applied to (9) gives

$$w_t^* = \beta\theta_w E_t\left\{w_{t+1}^*\right\} + (1-\beta\theta_w)\left(w_t - \left(1+\varepsilon_w\varphi\right)^{-1}\hat{\mu}_t^w\right) \qquad (10)$$

where $\hat{\mu}_t^w \equiv \mu_t^w - \mu^w$ denotes the deviation of the average wage markup from its steady state value μ^w and $\mu_t^w = (w_t - p_t) - mrs_t$. The log-linearised aggregate wage equation can then be derived as

$$w_t = \theta_w w_{t-1} + (1-\theta_w)w_t^* \qquad (11)$$

By combining (10) and (11) and using $\pi_t^w = w_t - w_{t-1}$ one obtains the wage inflation equation

$$\pi_t^w = \beta E_t\left\{\pi_{t+1}^w\right\} - \lambda_w\hat{\mu}_t^w \qquad (12)$$

where $\lambda_w = \dfrac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}$. The wage inflation equation is analogous to the price inflation equation (NKPC) in (5) and the interpretation of it is similar. When the desired markup is below the steady state level the wage inflation increases in order to catch up. In this extended model the wage inflation equation (12) replaces the condition $w_t - p_t = mrs_t$ in the basic model. As in the basic model there is an Euler equation for the consumers

$$c_t = E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - E_t\left\{\pi_{t+1}^p\right\} - \rho\right) \qquad (13)$$

where as before in the basic model $i_t = -\log Q_t$ is the yield on the one period bond.

Equilibrium

The analysis of equilibrium makes use of output gaps $\tilde{y}_t = y_t - y_t^n$ where the natural rate of output is the output that materializes in the absence of both sticky prices and sticky wages. The wage gap is defined as

$$\tilde{\varpi}_t = \varpi_t - \varpi_t^n$$

where $\varpi_t = w_t - p_t$ is the real wage rate and ϖ_t^n is the real wage in absence of sticky prices and wages, given by

$$\varpi_t^n = \log(1-\alpha) + \psi_{wa}^n a_t - \mu^p$$

where $\psi_{wa}^n \equiv \dfrac{1-\alpha\psi_{ya}^n}{1-\alpha}$ and $\psi_{ya}^n \equiv \dfrac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$, the latter already derived in the basic model.

We can now use $\mu_t^p = mpn_t - \varpi_t$ to get the price markup gap

$$\begin{aligned}\hat{\mu}_t^p &= (mpn_t - \varpi_t) - \mu^p \\ &= (\tilde{y}_t - \tilde{n}_t) - \tilde{\varpi}_t \\ &= -\frac{\alpha}{1-\alpha}\tilde{y}_t - \tilde{\varpi}_t\end{aligned} \quad (14)$$

Combining the previous price inflation equation with (14) gives

$$\pi_t^p = \beta E_t\left\{\pi_{t+1}^p\right\} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\varpi}_t \quad (15)$$

where $\kappa_p \equiv \dfrac{\alpha\lambda_p}{1-\alpha}$. Similarly, we can calculate

$$\begin{aligned}\hat{\mu}_t^w &= \varpi_t - mrs_t - \mu^w \\ &= \tilde{\varpi}_t - \left(\sigma\tilde{y}_t + \varphi\tilde{n}_t\right) \\ &= \tilde{\varpi}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right)\tilde{y}_t\end{aligned} \quad (16)$$

for the wage markup and by combining (12) and (16) get

$$\pi_t^w = \beta E_t\left\{\pi_{t+1}^w\right\} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\varpi}_t \quad (17)$$

where $\kappa_w = \lambda_w\left(\sigma + \dfrac{\varphi}{1-\alpha}\right)$. In addition there is an identity relating the changes in the wage gap to price and wage inflation and the natural real wage given by

$$\tilde{\varpi}_t \equiv \tilde{\varpi}_{t-1} + \pi_t^w - \pi_t^p - \Delta\varpi_t^n \quad (18)$$

The dynamic IS equation is now derived as

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\left\{\pi_{t+1}^p\right\} - r_t^n\right) + E_t\left\{\tilde{y}_{t+1}\right\} \quad (19)$$

where the natural rate of interest rate $r_t^n \equiv \rho + \sigma E_t\left\{\Delta y_t^n\right\}$ is the rate in an equilibrium with flexible prices and wages. Finally, the model is closed by formulating an interest rule

$$i_t = \rho + \phi_p \pi_t^p + \phi_w \pi_t^w + \phi_y \tilde{y}_t + v_t \quad (20)$$

where v_t is an disturbance term. (15) – (20) forms a dynamic system which may or may not have a unique solution. (Gali, J. 2008) shows that this system in general has no unique solution. However, restrictions can be imposed on the interest rate rule (20) that guarantees a unique solution. Provided $\phi_y = 0$ the condition $\phi_p + \phi_w > 1$ guarantees a unique solution $\tilde{y}_t = \pi_t^p = \pi_t^w = 0$ for all t. Diagram 1 shows the determinacy regions in that case.

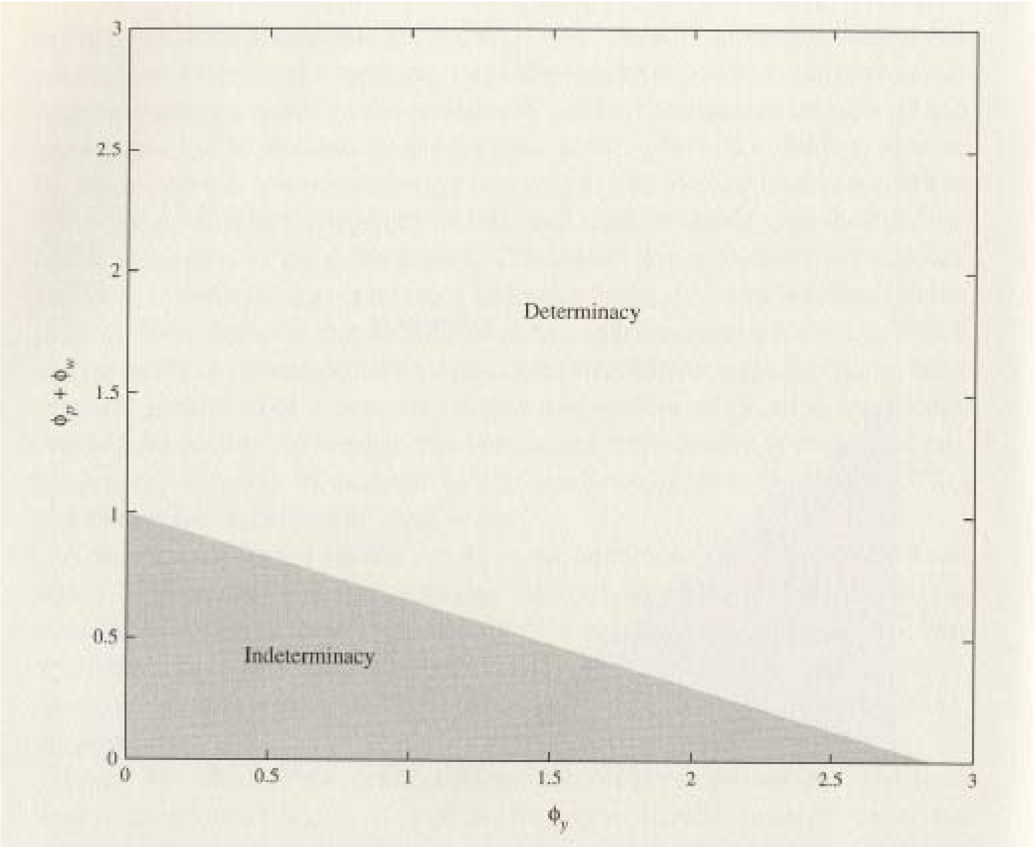

Diagram 1. Regions of determinacy and indeterminacy when $\phi_y = 0$. Source: (Gali, J. 2008).

(Gali, J. 2008) also studies the effects of monetary policy shocks, i.e. shocks to the interest rate rule in (20). Gali assumes $\phi_p = 1.5$ but $\phi_w = \phi_y = 0$ so that only the effects of the sticky wages are studied. He also assumes $\theta_p = 2/3$ and $\theta_w = 3/4$. In diagram 2 the effects of the shock, which is an 0.25 percentage points increase in the exogenous component of the interest rate rule, v_t, which in the absence of endogenous components in the rule would correspond to a one percentage point increase in the annualized nominal interest rate. The solid line shows the response when both wage and prices are sticky and the dashed lines when prices or wages are sticky, respectively. Obviously then, wage inflation shows a large impact when wages are flexible and price inflation when prices are flexible. However, the most realistic response seems to be realized when both prices and wages are flexible.

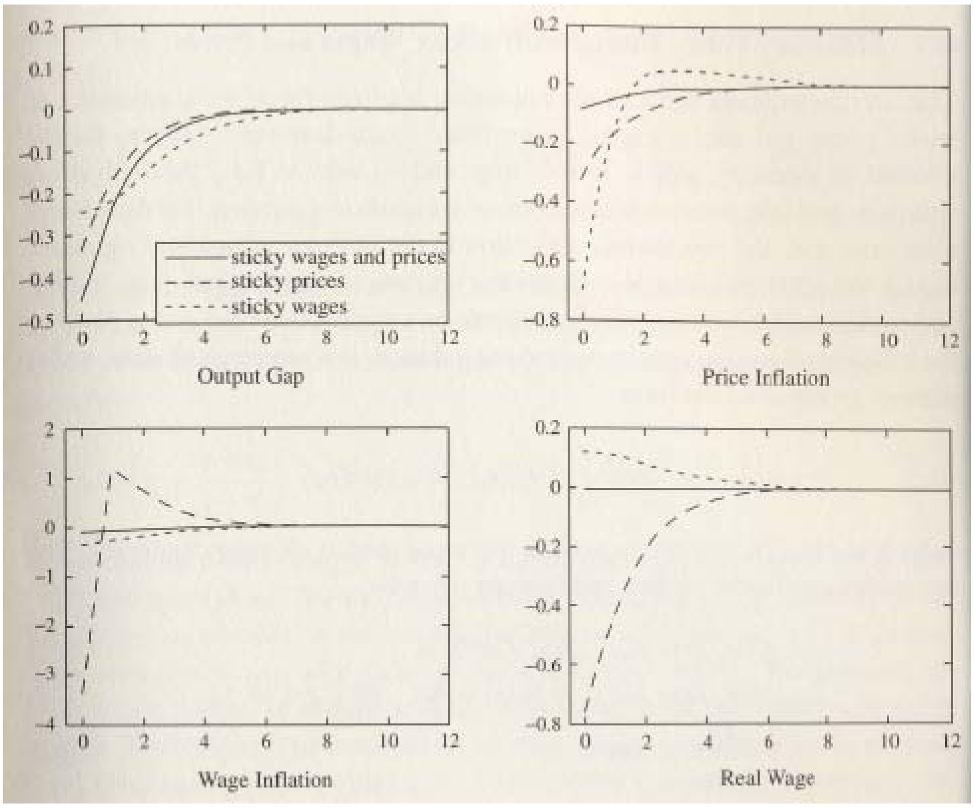

Diagram 2. Sticky prices and/or wages and the effects of monetary policy shocks. Source: (Gali, J. 2008).

Monetary policy with sticky prices and wages

A policy that seeks to stabilize prices only is suboptimal when wages are sticky

Some special cases

Simulations with simple rules

1)

References

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