The Basic Classical Model

The new Keynesian macro model is based on microeconomic foundations, i.e. optimizing household and firms. The shape of various functions, such as the IS or the Phillips curve, can be approximately the same as in the conventional IS/LM/AD model, but are derived explicitly from micro foundations. Let us describe the basic new Keynesian model. I do that without solving the model explicitly, but rather by describing the optimizing problem and the solution. I will follow (Richard Clarida et al., 1999) here and a similar description is given in (Jordi Gali, 2008) and the student interested in more details may read that book or consult other references found here.

The new Keynesian model – referred to as the workhorse model of today – results from research in various areas. The old Keynesian models were felt inadequate by many economists in the late 60s and beginning of the 70s, both due to theoretical deficiencies and to poor forecasting performance. The deficiencies could be listed:

- ✓ poor microeconomic foundations
    - o no coherent micro foundations
    - o only piecewise micro foundations
    - o inconsistent expectations
    - o business cycles based on price rigidities, poor foundations
    - o price and wage rigidities ad hoc
- ✓ poor econometrics
    - o Lucas critique
    - o expectations and econometrics
    - o cross-equation restrictions
    - o poor forecasting performance

The basic new Keynesian model is the result from research in these areas. Though theoretically satisfactory and coherent, the basic model presented here is very simple and make many strong simplifying assumptions.

It should be noted that the new Keynesian model is a modification of the real business cycle (RBC) model developed by (Finn E. Kydland and Edward C. Prescott, 1982, Edward C. Prescott, 1986) and others. They presented an alternative business cycle theory in which business cycles were generated by sequences of productivity shocks in models based on micro foundations. This was contrary to the Keynesian theory in which demand shocks and rigidities in wages and prices generated the cycles. The new Keynesian model extends the RBC model to include rigidities in wages and prices, based on some form of microeconomic behavior of non-competitive firms.

The basic model

Households

Households maximize the at time 0 expected discounted future utility of consumption and leisure

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where C_t is a consumption index given by

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

and $C_t(i)$ is the quantity consumed of good i at time t. N_t is the number of hours worked and β the discount rate or time preference. The marginal utility of consumption is denoted $U_{c,t} > 0$ and decreasing while $U_{n,t} < 0$ and decreasing, i.e. the marginal utility of leisure is positive. The budget constraint is

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{2}$$

where $P_t(i)$ is the price of good I, B_t the number of one-period bonds and Q_t the price of such a bond. T_t is a lump-sum tax or subsidy. The consumer takes all prices as given.

The solution to the problem of optimizing (1) subject to (2) is the isoelastic demand functions

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$$

where ε is the price elasticity, identical across goods, and where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$ is the index for the aggregate price level.

Aggregating over goods the aggregate budget constraint is obtained as

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{3}$$

In addition to allocating consumption among the many different goods, the consumer decides upon how much to spend or save and how much labor to supply. The first order conditions (FOC) for optimum can be obtained as

$$\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$
$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \qquad (4)$$

i.e. the marginal rate of substitution between consumption and leisure should be equal to the price ratio or relative price of consumption in terms of leisure.

These first order conditions can be obtained from a simple variational argument starting from the fact that utility maximization implies

$$U_{c,t} dC_t + U_{n,t} dN_t = 0$$

for any pair (dC_t, dN_t) satisfying the budget constraint

$$P_t dC_t = W_t dN_t$$

The first equation says that the marginal utility of a small increase in consumption is equal to the marginal disutility of a small increase in working hours. Otherwise it would be possible to increase utility by changing the consumption/leisure choice. The second equation says that the consumer must work more in order to consume more; that is the budget constraint. Rewrite these two equations to obtain the first FOC.

Similarly, consider

$$U_{c,t} dC_t + \beta E_t \left\{ U_{c,t+1} dC_{t+1} \right\} = 0$$

for any pair (C_t, C_{t+1}) satisfying the constraint $P_{t+1} dC_{t+1} = -\frac{P_t}{Q_t} dC_t$, showing the increase in consumption in period t+1 - $P_{t+1} dC_{t+1}$ - made possible by the savings in period t - $-\frac{P_t}{Q_t} dC_t$. The first equation says that the marginal utility of a an additional unit of consumption in period t must be equal to the discounted expected marginal utility of a decrease in consumption in period t+1. Otherwise, the consumer could increase utility for instance by reducing consumption in period t and increasing it in period t+1. Rewriting these two equations leads to the second FOC above.

Assuming now a specific period utility function

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \qquad (5)$$

the first order conditions become

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$$
$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{6}$$

which by log-linearisation becomes[1]

$$w_t - p_t = \sigma c_t + \varphi n_t$$
$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma}\left(i_i - E_t \{\pi_{t+1}\} - \rho\right) \tag{7}$$

where $\rho = -\log \beta$ and $i_t = -\log Q_t$ is the nominal interest rate. Note also that the following money demand function can be assumed

$$m_t - p_t = y_t - \eta i_i \tag{8}$$

from which the money supply can be derived. Note also, that there is no micro foundation for this equation. However, it has been shown the same form can be derived from different assumptions about how real balances deliver utility to the consumer, such that optimal level of real balances

Firms

Each firm has a production function

$$Y_t(i) = A_t N_t(i)^{1-\sigma} \tag{9}$$

i.e. labor is the only production factor and productivity is measured by A_t, which is exogenous and common to all firms. All firms take the aggregate price level and aggregate consumption index C_t as given.

Each period the firm maximizes profits

$$P_t Y_t - W_t N_t \tag{10}$$

subject to the production function taking wages and prices as given when in perfect competition. The FOC is

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[1] For a derivation, see Appendix. Note that $i_t = -\log Q_t$. This is so since $Q_t = \dfrac{1}{1+yield}$ is the price of one unit of a one period bond and hence $\log Q_t = 0 - \log(1+yield)$ and hence $-\log Q_t = \log(1+yield) \simeq yield$ and $i_t \equiv yield$ is a good approximation of the *nominal interest rate* for small yields.

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha} \tag{11}$$

i.e. the real wage equals the marginal product of labor, which can be log-linearised to

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha)$$

which is usually interpreted as the demand for labor.

Equilibrium

To derive the equilibrium in the classical model we use

$$y_t = c_t$$

and the log of the aggregate productions function

$$y_t = a_t + (1-\alpha) n_t$$

together with the FOC for the consumer (7) and for the firm (11) which with some calculations yields

$$\begin{aligned} n_t &= \psi_{na} a_t + \upsilon_n \\ y_t &= \psi_{ya} a_t + \upsilon_y \end{aligned}$$

where $\psi_{na} = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$, $\upsilon_n = \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$, $\psi_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$, $\upsilon_y = (1-\alpha)\upsilon_n$

Using the Euler equation for the consumption and the definition of the real interest rate $r_t \equiv i_t - E_t\{\pi_{t+1}\}$ we get

$$\begin{aligned} r_t &= \rho + \sigma E_t\{\Delta y_{t+1}\} \\ &= \rho + \sigma \psi_{ya} E_t\{\Delta a_{t+1}\} \end{aligned}$$

and the real wage rate

$$\begin{aligned} w_t - p_t &= a_t - \alpha n_t + \log(1-\alpha) \\ &= \psi_{wa} a_t + \upsilon_w \end{aligned}$$

where $\psi_{wa} = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\upsilon_w = \frac{\left(\sigma(1-\alpha)+\varphi\right)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$

Comments:

Interpret the parameters

$\beta < 1 \simeq 0.99$
$\rho \equiv -\log \beta \simeq 0.01$
$\sigma \simeq 1$
$\varphi \simeq 1$
$\alpha \simeq 0.33$
$\varepsilon = 1.5$
$\psi_{na} = 0$
$\upsilon_n = -0.2$
$\psi_{ya} = 1$
$\upsilon_y = -0.13$

would be fairly typical values for a logarithmic utility function $\sigma = 1$. This makes employment independent of productivity, which depends on $\sigma = 1$. $\psi_{wa} = 1$ which implies that real wages follow productivity. When $\sigma < 1$ real wages rise less than productivity and the substitution effect on labor supply resulting from a higher real wage dominates the negative effect caused by a smaller marginal utility of consumption, leading to an increase in employment. For instance, if $\sigma = 0.8$ then $\psi_{na} = 0.42$ which means that a 1 percent increase in productivity increases employment by 0.42 percent. If $\sigma = 1.2$ then $\psi_{na} = -0.1$ and a 1 percent increase in productivity *decreases* employment by 0.1 percent.

This would give a real interest rate

$$r_t = \rho + \sigma \psi_{ya} E_t \left\{ \Delta a_{t+1} \right\} \simeq 0.01 + 1 \cdot 1 \cdot 0.03 = 0.04$$

In this model there is independence between the real and nominal sectors. Taking the money supply into account through the equation

$$m_t - p_t = y_t - \eta i_t$$

nominal variables can be determined once a rule is specified for the monetary authority. It could be a rule for the money supply or for the nominal interest rate.

This is the classical/RBC setup. We now turn to the new Keynesian case, where prices are sticky.

Optimal real money balances

In the basic classical model there is monetary neutrality and no real effects of monetary policy. Hence, there is no welfare effects of monetary policy, which determines the nominal price levels. One possibility is to introduce real balances into the utility function, i.e. maximize

$$U\left(C_t, N_t, \frac{M_t}{P_t} \right) \tag{12}$$

subject to the constraint $C_t = A_t N_t^{1-\alpha}$. The FOCs are

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha)A_t N_t^{-\alpha}$$
$$U_{m,t} = 0 \qquad (13)$$

where the additional condition states that the private marginal utility of money should be equal to the social marginal cost, the cost of producing money, which is assumed to be zero. For the consumer there is the additional optimality condition

$$\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\} \qquad (15)$$

where the marginal utility of real money is positive. Again, consider a variational argument which requires

$$U_{c,t} dC_t + U_{m,t} dM_t = 0 \qquad (16)$$

for all (dC_t, dM_t) satisfying the constraint

$$P_t dC_t + (1-Q_t) dM_t = 0 \qquad (17)$$

Combining (16) and (17) and using the definition $i_t = -\log Q_t$ yields (15). But then the optimality condition $U_{m,t} = 0$ can only be met if $i_t = 0$. This policy is known as the Friedman's rule, see (Milton Friedman, 1969). This also implies a steady state inflation rate

$$\begin{aligned} \pi &= -(\rho + \sigma\gamma) \\ &= -r \end{aligned} \qquad (18)$$

Appendix: Log linearization of first-order condition

We obtained the FOC for the consumer

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$

which can be written

$$1 = Q_t^{-1} \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$

which can be rewritten as

$$1 = E_t \left\{ Q_t^{-1} \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$

and

$$1 = E_t \left\{ \exp\left(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho \right) \right\}$$

where $i_t = -\log Q_t$ and $\rho = -\log \beta$. In a perfect foresight steady state with constant inflation π and constant growth γ we have

$$i = \rho + \pi + \sigma\gamma$$

with the real rate of interest given by

$$\begin{aligned} r &\equiv i - \pi \\ &= \rho + \sigma\gamma \end{aligned}$$

where $\rho = -\log \beta$ is the discount rate. A first-order Taylor expansion of $\exp\left(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho \right)$

around this steady state yields

$$\begin{aligned} \exp\left(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho \right) &\simeq 1 + (i_t - i) - \sigma(\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi) \\ &= 1 + i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho \end{aligned}$$

which can be used in $1 = E_t \left\{ \exp\left(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho \right) \right\}$

to give

$$c_t = E_t \left\{ c_{t+1} \right\} - \frac{1}{\sigma} \left(i_t - E_t \left\{ \pi_{t+1} \right\} - \rho \right)$$

where the last bracket is the difference between the real interest and the discount rate. If these are equal we have $c_t = E_t\left\{c_{t+1}\right\}$ which means that consumption this period will be the same as the next period unless something unexpected happens. In steady state we have

$$c_t = E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left(\rho + \sigma\gamma - \rho\right)$$

$$c_{t+1} - c_t = \gamma$$

References

Ball, Laurence and Romer, David. "Sticky Prices as Coordination Failure." *American Economic Review*, 1991, *81*(3), pp. 539-52.

Blanchard, Olivier Jean and Kiyotaki, Nobuhiro. "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review*, 1987, *77*(4), pp. 647-66.

Calvo, Guillermo A. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 1983, *12*(3), pp. 383-98.

Carlin, Wendy and Soskice, David. *Macroeconomics: Imperfections, Institutions, and Policies*. Oxford: Oxford University Press, 2006.

Clarida, Richard; Gali, Jordi and Gertler, Mark. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature*, 1999, *37*(4), pp. 1661-707.

Friedman, Milton. *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine Press, 1969.

Gali, Jordi. *Monetary Policy, Inflation, and the Business Cycle*. Princeton: Princeton University Press, 2008.

Kydland, Finn E. and Prescott, Edward C. "Time to Build and Aggregate Fluctuations." *Econometrica*, 1982, *50*(6), pp. 1345-70.

Mankiw, N. Gregory and Reis, Ricardo. "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics*, 2002, *117*(4), pp. 1295-328.

Prescott, Edward C. "Theory Ahead of Business Cycle Measurement." *Federal Reserve Bank of Minneapolis Quarterly Review*, 1986, *10*(4), pp. 9-22.