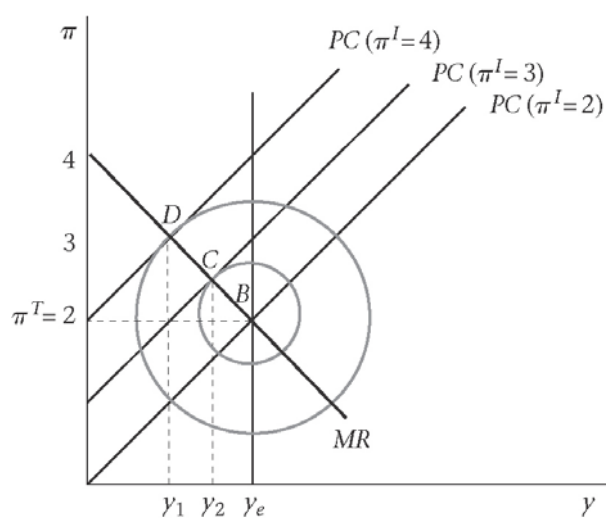


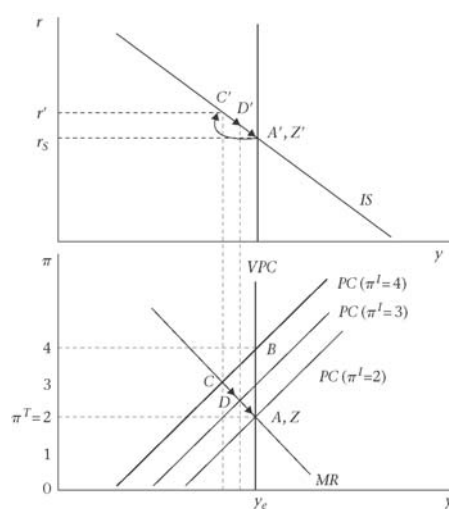
## Previous lecture

- IS/LM model discretionary changes in policy
- Rational expectations – all information
- Behavior of policy makers similar to other agents
- Possible to describe policy with rules
- This particular model: similar to Svensson (1997), influential paper

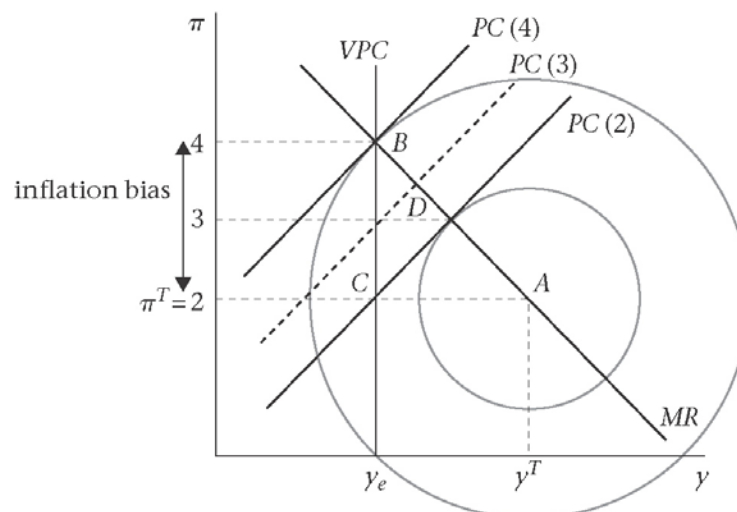
## The MR curve



### 3-equation model



### Illustration – the inflation bias



## Difference from IS/LM model

- Optimizing central bank => policy rule, not discretion
- Policy rule depends on price rigidity & central bank preferences
- Policy should commit to a realistic real target

## ”Equilibrium without microfoundations”

- Piecemeal microfoundations
  - Consumption and investment functions with microfoundations
- Critique from new classical economists
  - Kydland & Prescott
  - Lucas, Lucas critique, parameters not invariant to policy changes
- New research program
  - RBC new classical economists
  - Keynesian search for microfoundations for sticky prices

## What's good with microfoundations?

- Micro foundations but macro data
  - Representative agent (consumer, firm), not necessarily in new models
  - Aggregation problems
- Identification
  - Deep parameters for preferences/technology hard to identify
  - Calibration, new empirical methods, compare model predictions with data
- Previous empirical studies
  - Demand systems, consumption functions
    - from macro to micro studies

## Some different views

- Macroeconomics is empirical, related to policy
  - Start from observations, not from axioms
- Relationships are aggregated from micro level
- General vs partial equilibrium
  - General models must be simplified
- Coordination between different agents

## New synthesis

- RBC classical model & sticky prices
  - dynamic optimization
  - rational expectations
  - policy rules
  - sticky prices

## Basic new Classical(Keynesian) model

- Classical model with flexible prices
- Optimizing households and firms
- Forward-looking
- Central bank with policy rule
- Rational expectations
- (Sticky prices)

## Households 1

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad \text{max expected discounted utility}$$

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t \quad \text{budget constraint}$$

$Q_t$

Price in t for a one-period bond that delivers  
1 krona in period t+1

$$Q_t = \frac{1}{1 + \text{yield}}$$

## Households 2

$$\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1} P_t}{U_{c,t} P_{t+1}} \right\}$$

first order conditions for  
max utility (FOC)

## Simple reasoning

$$U_{c,t}dC_t + U_{n,t}dN_t = 0$$

small variation in consumption and leisure

$$P_t dC_t = W_t dN_t$$

constraint

$$U_{c,t}dC_t + \beta E_t \{U_{c,t+1}dC_{t+1}\} = 0$$

small variation between two periods

$$P_{t+1}dC_{t+1} = -\frac{P_t}{Q_t}dC_t$$

constraint

$$= -P_t(1 + \text{yield})dC_t$$

## Households 3

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

specific utility function

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi$$

FOC

$$Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$

FOC

$$w_t - p_t = \sigma c_t + \varphi n_t$$

FOC log-linearised labor supply

$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - \rho)$$

Log linear approximation around steady state

$$1 = Q^{-1} \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$

$$= E_t \{ \exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho) \} \quad \rho = -\log \beta$$

$$Q_t = \frac{1}{1 + \text{yield}} \quad \text{price of one unit of a one period bond}$$

$$\log Q_t = 0 - \log(1 + \text{yield})$$

$$-\log Q_t = \log(1 + \text{yield}) \approx \text{yield}$$

$$i_t = -\log Q_t \quad \text{nominal interest rate}$$

$$c_t = E_t \{ c_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho)$$

Approximation around steady state with  
(zero) growth and inflation

## Firms

$$Y_t = A_t N_t^{1-\sigma} \quad \text{production function}$$

$$\text{maximize} \quad P_t Y_t - W_t N_t$$

$$\frac{W_t}{P_t} = (1 - \alpha) A_t N_t^{-\alpha} \quad \text{FOC}$$

$$w_t - p_t = a_t - \alpha n_t + \log(1 - \alpha) \quad \text{log-linearised labor demand}$$



## Equilibrium 1

$$y_t = c_t \quad \text{equilibrium in goods market}$$

$$y_t = a_t + (1 - \alpha)n_t \quad \text{production function}$$

$$w_t - p_t = \alpha c_t + \varphi n_t \quad \text{FOC consumers}$$

$$w_t - p_t = a_t - \alpha n_t + \log(1 - \alpha) \quad \text{FOC firms}$$

## Equilibrium 2

$$\begin{aligned} n_t &= \psi_{na} a_t + v_n \\ y_t &= \psi_{ya} a_t + v_y \end{aligned} \quad \text{employment and output}$$

$$\psi_{na} = \frac{1 - \sigma}{\sigma(1 - \alpha) + \varphi + \alpha} \quad v_n = \frac{\log(1 - \alpha)}{\sigma(1 - \alpha) + \varphi + \alpha}$$

$$\psi_{ya} = \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} \quad v_y = (1 - \alpha)v_n$$

## Equilibrium 3

$$r_t \equiv i_t - E_t \{ \pi_{t+1} \} \quad \text{def real interest rate}$$

$$c_t = E_t \{ c_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho)$$

$$\begin{aligned} r_t &= \rho + \sigma E_t \{ \Delta y_{t+1} \} \\ &= \rho + \sigma \psi_{ya} E_t \{ \Delta a_{t+1} \} \end{aligned} \quad \begin{array}{l} \text{from consumption Euler equation} \\ \text{use definition of real interest rate,} \\ \text{the } y=c \text{ equilibrium in goods market,} \\ \text{and the equilibrium equation for } y \end{array}$$

## Equilibrium 4

$$\begin{aligned} w_t - p_t &= a_t - \alpha n_t + \log(1 - \alpha) \\ &= \psi_{wa} a_t + v_w \end{aligned} \quad \text{equilibrium wage rate}$$

$$\psi_{wa} = \frac{\sigma + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} \quad v_w = \frac{(\sigma(1 - \alpha) + \varphi) \log(1 - \alpha)}{\sigma(1 - \alpha) + \varphi + \alpha}$$

$$p_t = w_t - \psi_{wa} a_t + v_w$$

## Interpretation

- output, employment, the real wage rate, and the real interest rate are all driven by the exogenous productivity process
- the equilibrium of these real variables are independent of inflation and other nominal variables
- nominal variables are instead determined by monetary policy

## Money

$$m_t - p_t = y_t - \eta i_t \quad \text{money demand } ad \text{ hoc}$$

$$\max U \left( C_t, \frac{M_t}{P_t}, N_t \right) \quad \text{money in the utility function}$$

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha)A_t N_t^{-\alpha} \quad \text{FOCs}$$

$$U_{m,t} = 0$$

$$i_t = 0 \text{ \& } \pi_t = -r_t \quad \text{optimal policy, private cost and social cost are equal, i.e. zero}$$

## Parameters?

$$\beta < 1 \approx 0.99$$

$$\rho \equiv -\log \beta \approx 0.01$$

$$\sigma \approx 1$$

$$\varphi \approx 1$$

$$\alpha \approx 0.33$$

$$\varepsilon = 1.5$$

$$\psi_{na} = 0$$

$$\nu_n = -0.2$$

$$\psi_{ya} = 1$$

$$\nu_y = -0.13$$

## Implications

$\sigma = 1$       employment independent of productivity

$\psi_{wa} = 1$       real wages follow productivity

$\sigma = 0.8$       1 percent increase in productivity  $\Rightarrow$  +0.4 in employment

$\sigma = 1.2$       1 percent increase in productivity  $\Rightarrow$  -0.1 in employment

## Properties of classical model

- Classical dichotomy
- Always efficient equilibrium
- Inflation does not matter in basic model
- Money in the utility function
  - Zero nominal interest rate
  - Inflation = minus real interest rate
- Productivity the driving force

## Keynesian perspective

- Sticky prices
  - Takes time to adjust prices
  - Disequilibrium, not in steady state
- Market form
  - Non-competitive
  - Why not adjust prices?
    - Adjustment cost
    - Attention/information

## Non-competitive markets

- Welfare improvements can be achieved through policy
- Steady state output level too low
- Cooperation => non-competitive, e.g. bilateral monopoly => lower transaction costs => welfare?

## Sticky prices

- Practice
  - Empirical studies
    - Time series models
    - Structural models
    - Survey methods
    - Interviews
- Theory
  - Models
    - Time dependent
    - State dependent

## Practice

- Main results from different studies, vary somewhat but some robust:
- Time or state dependent pricing
  - Both very common
- Implicit and explicit contracts
- Low variability in costs
- Differences across goods
  - goods/services
  - homogenous/heterogenous
  - producer/consumer
- Downward/upward rigidity no big differences
- Menu costs not so important
- Kinked demand curve (fear to loose customers)

## Frequency of price changes

- Euro
  - CPI average duration 13 months
- Sweden
  - 27% change less than once a year
- USA
  - 50% of prices last less then 5.5 months (Bils and Klenow)
  - Blinder: Median firm once a year
- Prices more sticky in Europe than in the US

## Aggregate price rigidity

$$V \equiv P \cdot Y$$

$$v \equiv p + y$$

Define  $y^*$   $\equiv$  trend output

Define  $\hat{y} \equiv y - y^*$  output gap

Define  $\hat{v} \equiv v - y^*$  nominal shock

$$p = \alpha \hat{v} \rightarrow \hat{y} = (1 - \alpha) \hat{v}$$

$$p_t = \sum_{i=0}^n \alpha_i \hat{v}_{t-i} + \varepsilon_t$$

## $\alpha$ and the effects of shocks

- High inflation => shorter contracts => more flexible prices
- Low inflation regimes 1990s and onwards => longer contracts
- Real effects of monetary shocks larger in the later period



Country	$\alpha$ first five quarters	$\alpha$ second year	$\alpha$ after two years
Austria 1972:2 – 1989:4	0.612	0.340	0.952
Belgium 1963:2 – 1989:4	0.898	0.094	0.992
Canada 1963:2 – 1989:4	0.689	0.274	0.963
Denmark 1979:2 – 1989:4	0.535	0.116	0.651
Finland 1977:2 – 1989:4	0.616	0.410	1.026
France 1967:2 – 1989:4	0.724	0.191	0.915
Germany 1967:2 – 1989:4	0.504	0.307	0.811
Greece 1963:2 - 1989:4	1.026	0.000	1.026
Ireland 1963:2 – 1989:4	0.939	0.050	0.989
Italy 1972:2 – 1989:4	0.911	0.115	1.026
Japan 1967:2 – 1989:4	0.988	0.059	1.047
Netherlands 1979:2 – 1989:4	0.784	0.083	0.867
Portugal 1963:2 – 1999:1	0.873	0.085	0.958
Spain 1972:2 – 1989:4	0.952	0.156	1.108
Sweden 1963:2 – 1989:4	0.681	0.301	0.982
United Kingdom 1965:2 – 1989:4	1.013	-0.030	0.983
USA 1963:2 – 1989:4	0.443	0.428	0.871

Country	$\alpha$ first five quarters	$\alpha$ second year	$\alpha$ after two years
Austria 1990:1 – 1999:1	0.703	0.422	1.125
Belgium 1990:1 – 1998:4	0.321	0.281	0.603
Canada 1990:1 – 1999:1	0.358	0.446	0.804
Denmark 1990:1 – 1999:1	0.665	0.323	0.989
Finland 1990:1 – 1998:4	0.266	0.283	0.549
France 1990:1 – 1999:1	0.293	0.384	0.677
Germany 1990:1 – 1999:1	0.486	0.294	0.780
Greece 1990:1 - 1998:4	0.624	0.540	1.164
Ireland 1990:1 – 1998:4	0.083	-0.513	-0.429
Italy 1990:1 – 1999:1	0.688	0.307	0.995
Japan 1990:1 – 1999:1	0.192	0.671	0.863
Netherlands 1990:1 – 1998:4	0.143	0.160	0.303
Portugal 1990:1 – 1999:1	0.668	0.239	0.907
Spain 1990:1 – 1999:1	0.888	0.187	1.075
Sweden 1990:1 – 1998:4	0.401	0.390	0.791
United Kingdom 1990:1 – 1999:1	0.458	0.744	1.202
USA 1990:1 – 1999:1	0.701	0.474	1.175

## Empirical results

- The flexibility parameter for the first 5 quarters varies between 0.5 and 1 and for the second year (quarters 6-9) between 0 and 0.4
- The price adjustment is approximately complete ( ) after two years for almost all countries
- prices are more flexible in the earlier than in the later period for most of the countries

## Consequences

- Contract length in low inflation regime
- Monetary shocks more important for real variables in low inflation regime
- Increased stickiness => relative price dispersion => resource allocation

## Sticky price

- Benefits
  - Customer relationships
    - Heterogenous goods/adapted to buyers
    - Investments in customer relationships
    - Reduction in search costs
- External economies/diseconomies
  - Sticky price => aggregate price rigidity => resource allocation

## NKPC

- Basic new Keynesian model
  - Does not generate enough inflation persistence
  - This is accomplished by ad hoc persistence in the stochastic shocks

## Basic New Keynesian Model

- Consumer the same as in the classical model
- Monetary policy - different effects
- MP could possibly affect resource allocation if prices are sticky

- Imperfect competition
- Each firm produces a differentiated good
- Only some of the firms can change the price (optimize)

## Households

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad \text{max expected discounted utility}$$

$$C_t = \left( \int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{consumption index}$$

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \quad \text{budget constraint}$$

## Consumers' solution

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \quad \text{aggregate budget constraint}$$

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t \quad \text{demand functions}$$

## Firms

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad \text{production function}$$

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t \quad \text{demand functions}$$

$$\log C_t(i) = \log C_t - \varepsilon (\log P_t(i) - \log P_t) \quad \text{logarithmic}$$

$$c_t(i) = c_t - \varepsilon (p_t(i) - p_t)$$

## Calvo model

- Each firm may only reset their price with probability  $1-\theta$  in any given period
- $1-\theta$  is exogenous
- In each period a fraction  $1-\theta$  of the producers reset their prices
- a fraction  $\theta$  keep them unchanged

## Calvo 2

- average duration of a price is  $1/1-\theta$
- $\theta$  is a measure of price rigidity

## Calvo 3 aggregate price dynamics

$$P_t = \left[ \theta (P_{t-1})^{1-\varepsilon} + (1-\theta) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

$$\Pi_t = \frac{P_t}{P_{t-1}} \quad \text{definition}$$

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\varepsilon}$$

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \quad \text{logarithmic approximation around steady state with zero inflation}$$

## Optimal price?

- The optimizing firm will choose the price  $P_t^*$
- that maximizes the current market value of the discounted expected profits generated while that price remains effective (is not reset)

## Optimal price, problem

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( P_t^* Y_{t+k|t} - \psi_{t+k}(Y_{t+k|t}) \right) \right\} \quad \psi_{t+k} \text{ cost function}$$

subject to  $Y_{t+k|t}$  Output in t+k for firm that resets price at t

$$Y_{t+k|t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k}$$

$$Q_{t,t+k} \equiv \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right) \quad \text{discount factor for nominal payoffs}$$

$$\log Q_{t,t+k} = k \log \beta - \sigma \Delta y_{t+k} - \pi_{t+k} \quad k=0 \Rightarrow Q=1$$

$$\sigma = 1 \rightarrow \log Q_{t,t+k} = k \log \beta - [\Delta y_{t+k} + \pi_{t+k}]$$



## Solution

$$p_t^* = \underbrace{\mu}_{\text{markup}} + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \underbrace{mc_{t+k|t}}_{\text{nominal marginal cost}} + p_{t+k} \right\}$$

$$\mu = \log M = \log \left( \frac{\varepsilon}{\varepsilon - 1} \right) \quad \text{markup}$$

price is the desired markup over a weighted average of their current and discounted expected future nominal marginal costs, with the weights being proportional to the probability of the price remaining effective at each horizon

## Equilibrium

$$Y_t(i) = C_t(i)$$

$$Y_t = C_t$$

$$N_t = \int_0^1 N_t(i) di$$

## Equilibrium

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \lambda (mc_t - mc) \quad \text{Real marginal cost}$$

$$p_t^* = \mu_t + \log(\psi_{t+k|t})$$

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{(1-\alpha)}{(1-\alpha+\alpha\varepsilon)} \quad mc_t \equiv \log(\psi_{t+k|t}) - p_t$$

$$mc_t = -\mu_t$$

$$\lambda = \frac{(1-0.67)(1-0.99 \cdot 0.67)}{0.67} \frac{(1-0.33)}{(1-0.33+0.33 \cdot 3)} = 0.165$$

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \lambda [(\log \psi_t - \log \psi) - (p_t - p)]$$

## Interpretation

$$\pi_t = \beta E_t \{ \pi_{t+1} \} - \lambda (\mu_t - \mu)$$

- If markups are high (relative to steady state) then inflation is low
- Counterintuitive but depends on forward-looking behavior

## Interpretation

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k E_t \{ mc_{t+k} - mc \}$$

Expected real marginal cost high relative to steady state

Markups expected to be below steady state => inflation will be high => firms that reset price choose a price above the average price level

## Output gap

$$mc_t - mc = \left( \alpha + \frac{\varphi + \alpha}{1 - \alpha} \right) (y_t - y_t^n)$$

$$\pi_t = \gamma + \beta E_t \{ \pi_{t+1} \} + \kappa (y_t - y_t^n)$$

$$\kappa = \lambda \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$$

$$\kappa = 0.165 \left( 1 + \frac{1 + 0.33}{1 - 0.33} \right) = 0.49$$

## Dynamic IS equation

$$Y_t(i) = C_t(i)$$

$$Y_t = C_t$$

$$c_{t+1} = c_t + \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho) + u_{t+1}$$

$$c_{t+1} = E_t \{ c_{t+1} \} + u_{t+1}$$

remember interpretation

Rewrite the consumer's Euler equation in terms of the output gap

$$y_t = E_t \{ y_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho)$$

$$y_t - y_t^n = E_t \{ y_{t+1} - y_{t+1}^n \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho) + E_t \{ y_{t+1}^n - y_t^n \}$$

$$\tilde{y}_t = E_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - r_t^n)$$

$$r_t^n = \rho + \sigma E_t \{ \Delta y_{t+1}^n \}$$

Real rate of interest at flexible prices, keeps output at natural level

## Equilibrium

$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - \rho)$$

rewrite

$$i_t - E_t \{\pi_{t+1}\} = \rho + \sigma E_t \{\Delta y_{t+1}\}$$

$$\begin{aligned} r_t^n &= \rho + \sigma E_t \{\Delta y_{t+1}^n\} \\ &= \rho + \sigma \psi_{ya}^n E_t \{\Delta a_{t+1}\} \end{aligned}$$

$$y_t - y_t^n = -\frac{1}{\sigma} \sum_{k=0}^{\infty} (r_{t+k} - r_{t+k}^n)$$

that the output gap is proportional to the real interest rate gap, i.e. the difference between the real interest rate and the natural interest rate.

## Monetary policy

$$i_t = \rho + \phi_{\pi} \pi_t + \phi_y (y_t - y_t^n) + v_t$$

compare with 3-equation model of Svensson (1997)

Assume

$$\phi_{\pi} = 1.5 \text{ and } \phi_y = 0.125$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \quad \rho_v = 0.5$$

and

$$\beta < 1 \approx 0.99$$

$$\rho \equiv -\log \beta \approx 0.01$$

$$\sigma \approx 1$$

$$\varphi \approx 1$$

$$\alpha \approx 0.33$$

$$\varepsilon = 6$$

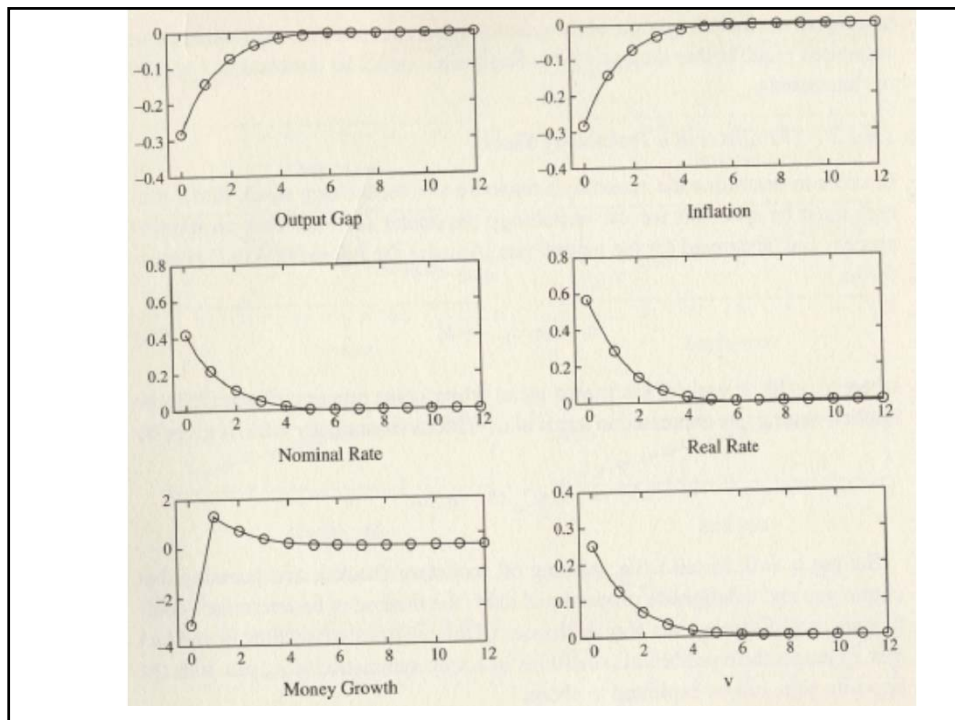
$$\psi_{na} = 0$$

$$\nu_n = -0.2$$

$$\psi_{ya} = 1$$

$$\nu_y = -0.13$$

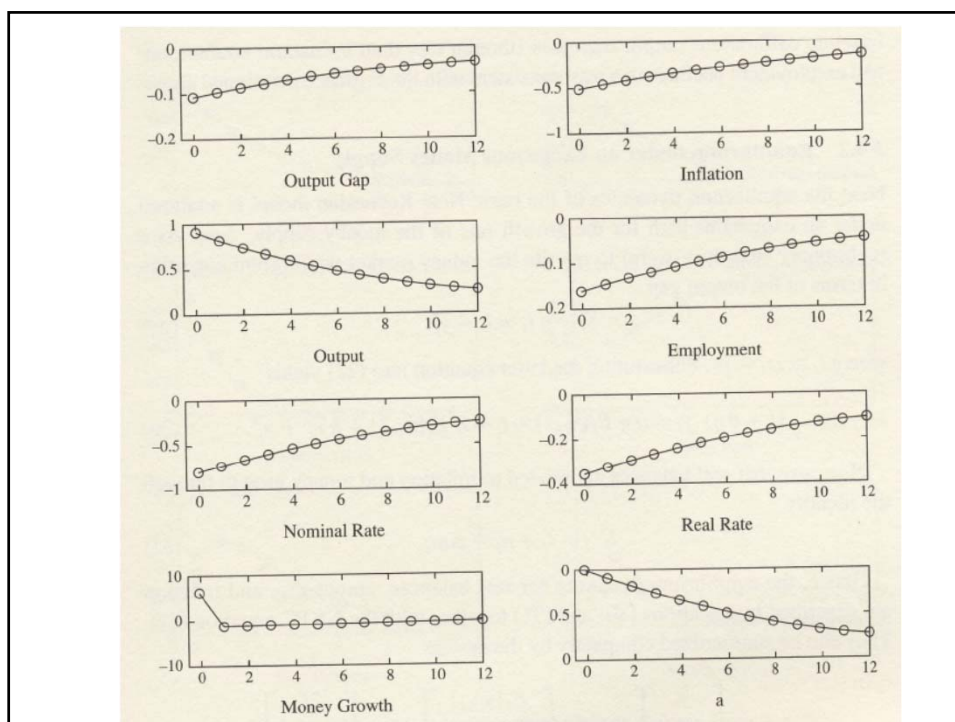
monetary policy shock



and technology shock

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

$$\rho_a = 0.9$$



## More on policy

- Optimal policy
  - Uniqueness
  - Realistic rules
    - Information requirements
      - Unobservables like flexible price output or interest rate
- Simple policy rules
  - Taylor rules
- More on this later